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ABSTRACT

The author investigates elementary school students' performance when solving selected open distributive sentences in relation to three factors (Open Sentence Type, Context, Number Size) and identifies and classifies solution methods attempted by students and students' errors in performance. Eighty fifth-grade students participated in the investigation. Each student was presented four open distributive sentences in one of four contexts with each sentence reflecting a different combination of four possible combinations of two levels of Open Sentence Type and two levels of Number Size. Performance of each student during an individual interview provided data via audio-taped verbal responses, interview tests, and a record of numerical solutions found for each sentence. Analysis of variance tests determined significance levels of the various independent variable effects. The author notes that the elementary school students' lack of using the distributive principle in this investigation echos the results of previous studies. He suggests that postponement of formal introduction to the distributive principle until students reach a higher level of mathematical maturity may be a wise decision and would also create implications for change in the rationalization and introduction of multiplication algorithms in the elementary schools. (Author/MN)

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TECHNICAL REPORT NO. 397

an analysis of fifth-grade students' performance when solving selected open distributive sentences

SEPTEMBER 1976

WISCONSIN RESEARCH AND DEVELOPMENT CENTER FOR COGNITIVE LEARNING

Technical Report No. 397

AN ANALYSIS OF FIFTH-GRADE STUDENTS' PERFORMANCE WHEN SOLVING SELECTED OPEN DISTRIBUTIVE SENTENCES

by

Charles Eugene Hobbs

Report from the Project on
Conditions of Learning and Instructional Strategies

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ABSTRACT

The Problem

The investigation sought to study elementary school students' performance when solving selected open distributive sentences in relation to three factors (Open Sentence Type, Context, Number Size), and to study students' use of the distributive principle as related to selected levels of their recognition of that distributive principle. Also, solution methods attempted by students and students' errors in performance were identified and classified.

Procedure

Student performance on a recognition level test was used to assign each of 301 fifth-grade students to a level of recognition of the distributive principle. Twenty students from each of four recognition level groups were selected to participate in the investigation. Each student was presented four open distributive sentences in one of four contexts with each sentence reflecting a different combination of four possible combinations of two levels of Open Sentence Type and two levels of Number Size. Data was provided by performance of each student during an individual interview.

Results

Since no significant difference was noted between boys' and girls' overall performance, findings are reported for the sexes combined.

Data supported lack of differential performance due to Open Sentence Type, Context, and/or interactions between three factors (Open Sentence Type, Context, Number Size) when dependent variables were number of correct solutions, number of times the distributive principle was used, and number of times the most frequently selected incorrect solution method was used. Data supported lack of differential performance due to Number Size when the dependent variables were number of correct solutions and number of times the distributive principle was used. Data did not support lack of differential performance due to Number Size when the dependent variable was number of times the most frequently selected incorrect solution method was used. Students selected the incorrect solution method more often when solving open distributive sentences containing small numbers than those containing large numbers.

Solution methods used by students were categorized as correct or incorrect. Two correct solution methods and a variety of incorrect solution methods were identified. Based on a total of 320 responses from 80 subjects, students selected to use the distributive principle 46 times (36 times correctly), while the other correct solution method was used a total of 30 times (17 times correctly). Due to students' "poor" performance and low reliability of the Recognition Level Test, the second purpose of the investigation was not achieved. With low variability of performance, data would tend to support no differential effects.

Errors were identified and categorized according to the following categories: Incorrect use of the distributive principle, Use of incorrect solution methods, Computational errors, and Difficulty in reading the open distributive sentences. Students used the distributive principle incorrectly 10 times, made a variety of errors in computation, and had considerable difficulty in correctly reading open distributive sentences. However, use of incorrect solution methods accounted for 242 of 265 incorrect solutions. One incorrect solution method was used 198 times and accounted for 74.7% of 265 incorrect solutions given. This incorrect solution method is characterized as follows: Given an open distributive sentence of the form (a x b) + (c x d) = (__ x __), the two indicated products are used as a solution pair.

In conclusion, elementary school students' lack of using the distributive principle in the present investigation echoes the results of Weaver (1973) and Hobbs (1973). Results do not indicate, however, if elementary school students do or do not have an informal "idea" about distributivity and cannot recognize and/or use this principle in the formal ways presented in the investigations. A major implication of these investigations is that the pedagogical reason for teaching the distributive principle may not hold true for elementary school students. The postponement of a formal introduction to the distributive principle until students reach a higher level of mathematical maturity may very well be a wise decision to make. If this postponement occurs, implications for a change in rationalization and introduction of multiplication algorithms in the elementary school must be considered.

J. Fred Weaver

Chapter I

INTRODUCTION

The importance (mathematically and pedagogically) of the introduction and use of fundamental principles of mathematics at the elementary school level is summarized by Van Engen (1966) as follows:

> The use of the fundamental principles of mathematics simplifies the teaching of arithmetic. This, of course, is an immediate "payoff" for introducing these ideas into arithmetic. However, another valid reason can be given. The fundamental principles of arithmetic should be taught to the elementary school child. . . . it follows as a matter of reason that they [fundamental principles] must serve as the foundation on which all of the elementary school arithmetic should be constructed [p. 316].

Mathematical and pedagogical importance indicate two reasons why modern mathematics curricula tend to introduce the elementary school student to experiences with fundamental principles of mathematics. Among these experiences is an introduction to the principle of distributivity. This principle is important as a "building block" in the construction of number systems rich in structure (e.g., the field of real numbers). This principle may be useful pedagogically in helping students understand the basic algorithm underlying the process of multiplying and other basic structural components of mathematics. The following examples illustrate how the distributive principle may be used in most contemporary elementary school mathematics programs in order to enhance students' understanding of the multiplication process.

Consider the calculation of $4 \times 17 = n$. Three ways of arriving at the solution are:

1. $4 \times 17 = 4 \times (10 + 7) = (4 \times 10) + (4 \times 7) = 40 + 28 = 68.$

2.
$$\begin{array}{r} 17 \\ \times 4 \\ \hline 28 \\ +40 \\ \hline 68 \end{array}$$

3.
$$\begin{array}{r} 17 \\ \times 4 \\ \hline 68 \end{array}$$

An understanding of the distributive principle and its application in solving the problem as illustrated in the first and second examples above is used by most current elementary school mathematics curricula in hopes of forming a basic understanding of the multiplication algorithm as indicated in the third example above.

The distributive principle may also be used in hopes of extending the knowledge of basic multiplication facts. For example, if a student knows $5 \times 3 = 15$ and $4 \times 3 = 12$ then open sentences of the following forms may lead to the result that $9 \times 3 = 27$.

1. $5 \times 3 = ___$ $\quad 4 \times 3 = ___$

   so $9 \times 3 = ___$

2. $(5 \times 3) + (4 \times 3) = ___ + ___ = ___$

   $(5 \times 3) + (4 \times 3) = ___ \times ___ = ___$

3. $5 \times 3 = 15$

   $4 \times 3 = 12$

   $9 \times 3 = __$

Many examples of sentences using such variations of the distributive principle, including commuted forms (e.g., (3 x 5) + (3 x 4) = ___ + ___ = ___) may be found in current elementary school mathematics curricula (e.g., Houghton-Mifflin, 1970, grade 3, page 283). The investigation presented in this report focuses on the performance of fifth-grade students in recognizing and using the distributive principle when solving certain open distributive sentences.

Crawford (1964) reported evidence indicating that the distributive principle is one of the most difficult field axioms for students to learn at grade levels extending beyond the elementary school level. Weaver's pilot study data (1972) and major study data (1973c) illustrate the difficulty elementary school students have when solving certain open sentences with at least one possible method of solution involving use of the distributive principle of multiplication over addition in a subset of the set of whole numbers. Weaver's studies indicate that students in grades 4-7 generally fail to use the distributive principle when solving open sentences of the form

$$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (__ \times __)$$

$$(\underline{b} \times \underline{a}) + (\underline{c} \times \underline{a}) = (__ \times __)$$

where $\underline{a}$, $\underline{b}$, and $\underline{c}$ are elements of a selected subset of whole numbers.[1]

The difficulty students have in solving open sentences of the form indicated above and in recognizing possible use of the distributive principle in solving these open sentences may be due to a variety of factors.

---

[1]Weaver's studies dealt with other open sentence forms also, but only the two forms indicated are considered in this investigation.

This investigation focuses on some of these factors with the fourfold purpose:

1. To investigate the relationship between elementary school students' performance when solving open distributive sentences of the form indicated above and selected levels of three factors (Open Sentence Type, Context, Number Size) associated with these open sentences (These factors and levels are characterized in Chapter II.),

2. To investigate the relationship between elementary school students' use of the distributive principle as a method for solving the open distributive sentences and selected levels of students' recognition of the distributive principle,

3. To identify and classify other methods attempted by elementary school students when solving the open distributive sentences, and

4. To identify and classify errors in elementary school students' performance when solving the open distributive sentences.

Most terms which occur throughout this report are to be interpreted as used in standardized mathematical language and some have already been used with that implicit interpretation throughout the previous discussion. However, certain selected terms are defined to prevent confusion concerning the author's intent. The characterization and illustration of terms reflect the use of the ordinary operations of multiplication and addition in the set of whole numbers in the elementary school and in the investigation being reported.

Operation: An operation is a way of associating an ordered pair of numbers with a unique third number.

Addition: Addition is a binary operation which assigns to an ordered pair of whole numbers a specified whole number; the components of the ordered pair are called addends, and the specified whole number is called the sum.

Multiplication: Multiplication is a binary operation which assigns to each ordered pair of whole numbers a specified whole number; the components of the ordered pair are called factors, and the specified whole number is called the product.

Distributive Principle: Generally, the distributive principle asserts that the product of a whole number and the sum of two whole numbers is equal to the sum of the products of the first whole number and each of the members of the sum. Symbolically this assertion may be presented as

$$a \times (b + c) = (a \times b) + (a \times c)$$

where a, b, and c represent whole numbers, x represents the operation of multiplication, and + represents the operation of addition. The use of the commutative property for multiplication and the symmetric property for the equality relation may be used to form other symbolic representations of the distributive principle as indicated by Figure 1.

Distributive Sentence: A distributive sentence is an equation illustrating the use, or possible use, of the distributive principle in the set of whole numbers.

Closed Sentence: An equation not containing a placeholder (blank, frame, etc.) is a closed sentence. Examples are

$$2 \times (3 + 4) = (2 \times 3) + (3 \times 4)$$

and

$$(5 \times 6) + (3 \times 6) = (5 + 3) \times 6.$$

Open Sentence: An equation containing at least one placeholder (blank, frame, etc.) is an open sentence. Examples are

$$(5 \times 6) + (3 \times 6) = (___ \times ___)$$

and

$$(3 \times 2) + (3 \times 4) = (___ \times ___).$$

Solution Set of an Open Sentence: The solution set of an open sentence is the set containing all elements of a specified domain which may be placed in the position of the placeholder(s) to make the sentence true and only those elements of the domain which make the sentence true. For example, the solution set of the open sentence

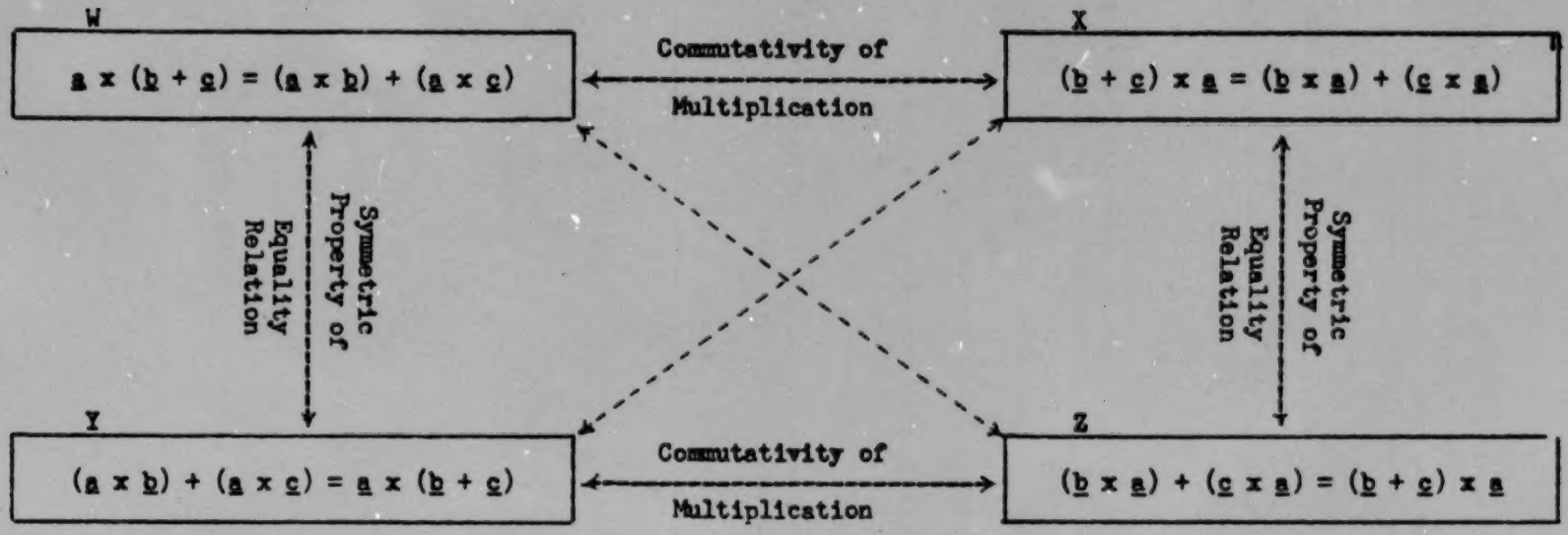

Fig. 1. Four equivalent variations of the distributivity of multiplication over addition for whole numbers a, b, c.[a]

Note: The variations of distributivity considered in this investigation are directly or analogously associated with Y and Z of Figure 1 rather than with W and X.

[a] Figure 1 is an adaptation of a similar diagram used by Weaver (1973c).

$(2 \times 3) + (2 \times 4) = (__ \times __)$

in the set of whole numbers is the set of ordered pairs

[(2,7); (7,2); (1,14); (14,1)].

This brief introduction provides the necessary foundation for a discussion of the fourfold purpose of this investigation and the research background from which the fourfold purpose was generated. Chapter II presents a sequential development of the specific purposes via a review of selected research; Chapter III presents the design of the investigation; Chapter IV presents the conduct of the investigation; Chapter V presents results of the investigation; and Chapter VI presents conclusions and implications of the investigation.

Chapter II

THE PROBLEM

The generation of the fourfold purpose (see Chapter I) of the investigation is presented in this chapter through a review of selected research. Many researchers in mathematics created and/or used tests containing items pertaining to distributivity but since their researches did not discuss or focus on distributivity to any great extent these researches are not reviewed (e.g., Hartlein, 1966).

A sequential review of selected research important in creating the background from which the fourfold purpose was generated is discussed. First, a number of investigations (e.g., Crawford, 1964; Gray, 1965; Schell, 1964) that have noted the occurrence of a variety of events indicating that elementary school students seem to have difficulty in understanding the distributive principle is presented. Second, investigations by Weaver (1971, 1973a) and Grouws (1971, 1972) that reported information concerning the relationship between elementary school students' performance in solving certain open sentences (non-distributive) and selected factors which could affect students' performance are presented. The results of these latter investigations indicated a need to continue the study of factors associated with elementary school students' performance in solving open sentences. Third, Weaver's investigation (1973c) that focused on the

relationship between elementary school students' performance in solving certain open distributive sentences and selected factors which could affect students' performance is presented. This investigation indicated a need for further study of elementary school students' performance in solving open distributive sentences and a need to study students' solution procedures and errors when solving open distributive sentences. The following discussion presents more details concerning each of the above investigations.

Crawford's Study

Crawford (1964) investigated the age-grade trends in students' understanding of field axioms in grades 4, 6, 8, 9, 10, and 12. A test constructed by Crawford was administered to 120 students at each of the selected grade levels. Among Crawford's findings was the following order of difficulty of the axioms from easiest to most difficult: commutativity, inverse, closure, identity, associativity, and distributivity. The difficulty index for each test item related to each axiom (three items for distributivity and six items for each of the remaining axioms) was determined. The difficulty indexes for items related to each axiom were summed and the sum for the distributive items doubled. The resulting numerical values were considered as the relative difficulty of each axiom. Crawford compared these values in arriving at what he terms as "suggestive data" on the order of difficulty of individual axioms. Crawford pointed out that it is important to recognize that this order of difficulty was based on only a few test items per axiom, that the order of difficulty was determined with no regard to

different operations (addition and multiplication), and that the pattern of difficulty may be different for students presented different mathematics programs (e.g., "modern" as opposed to "traditional"). However, Crawford's findings provide limited evidence that distributivity is one of the most difficult, if not the most difficult field axiom for students to grasp.

Schell's Study

Schell (1964) studied the learning of the distributive principle by 198 third-grade students as a part of a larger study of the learning of multiplication. A 41-item test with the last 14 items dealing with the distributive principle was administered to the 198 students following the presentation of nine lessons to the students. The first five lessons dealt with introduction of multiplication and the learning of basic facts; lessons 6 and 7 dealt with the distributive principle; lessons 8 and 9 reviewed the previous lessons. Tests from the low 27% and the top 27% of the students were selected for study. Almost 80% of the top group answered the 14 items on distributivity correctly and only 14% of the low group answered the 14 items correctly. Among Schell's (1968) reported conclusions were the following:

1. Pupils in grade three can learn to use the distributive principle of multiplication in only two introductory lessons and a review session even though their level of performance is not as high as on learning basic multiplication facts,

2. The statistical comparison of the proportion of correct responses to the distributive and non-distributive property items showed that the former were significantly more difficult than the latter, and

> 3. A comparison of the proportion of correct responses by the upper and lower 27% of pupils for the distributive and non-distributive property items showed that low-scoring pupils had a decidedly more difficult time with distributive than non-distributive property items while high-scoring pupils performed approximately equal on both types of items [p. 30].

Schell commented that if students are expected to use and understand the distributive principle at more than a minimum level of competency without additional instruction then the distributive principle should not be introduced until grade four.

In Schell's study of the learning of multiplication, two treatment groups were formed. One group (T_1) was introduced to multiplication through use of arrays and the second group (T_2) was introduced to multiplication through use of a variety approach. Schell did not report consideration of a possible difference in student performance due to the teaching of only the left distributive principle to one treatment group (T_1) and the teaching of only the right distributive principle to the second treatment group (T_2) in his investigation. A more detailed discussion of Schell's use of the left and the right distributive principle when teaching different treatment groups is presented later in this chapter. However, once again one finds evidence that the effect of a variety of factors on student performance in using and understanding the distributive principle has not been given proper attention.

The present investigation focuses on students' solving of open distributive sentences at the fifth-grade level after students have been exposed to the idea of distributivity for three years. Information concerning elementary school students' performance with a greater amount of

instruction will be gained. Student performance when presented open distributive sentences using the left distributive principle and when presented open distributive sentences using the right distributive principle will be studied in the present investigation.

Gray's Study

Gray (1964) investigated the effectiveness of a program of instruction in introductory multiplication based on the development of an understanding of the distributive principle at the third-grade level. Twenty-two classes of third-grade students (480 students total) participated in the investigation. Two treatment groups were formed and each group was presented 18 lessons introducing multiplication. The first five lessons were identical for each treatment group. The difference between the remaining lessons was the restriction of teaching the distributive principle to one group with care being made that the other group receive no introduction to the distributive principle. Fifty-five subjects were randomly selected from each treatment group and each student presented an interview test consisting of five multiplication items. One purpose of the interview test was to determine whether or not students would use the distributive principle to find products. Results of the interview tests were that 45.5% of the students who were taught the distributive principle method used the distributive principle for at least one of the five test items and only 18% used the distributive principle to find products for all five test items. This finding and other data[1] led Gray (1965) to conclude:

---

[1]The author means in no way to imply that the interview data were the only data upon which Gray bases the conclusions. Nor does the author wish to imply that the interview data were the only data furnished by Gray.

1. A program of arithmetic instruction which introduces multiplication by a method stressing understanding of the distributive principle produced results superior to methods currently in use.

2. Knowledge of the distributive principle appears to enable students to proceed independently in the solution of untaught multiplication combinations.

3. Students appear not to develop an understanding of the distributive principle unless it is specifically taught.

4. Insofar as the distributive principle is an element of the structure of mathematics, the findings tend to support the assumption that teaching for an understanding of structure can produce superior results in terms of student growth [p. 202].

Gray reports that students appear not to develop an understanding of the distributive principle unless it is specifically taught. Performance by students specifically taught the distributive principle was low as reflected by the interview test results. The present investigation will provide information concerning the development of the understanding and use of the distributive principle by students introduced to the distributive principle through regular mathematics programs.

CAA Study

Further evidence of the difficulty students seem to have in understanding the distributive principle at the elementary level is given by the Concept Attainment Abilities Project (CAA) of the Wisconsin Research and Development Center for Cognitive Learning (Romberg, Steitz, & Frayer, 1971). The distributive principle was checked for familiarity to fourth-grade students by the project and was considered by teachers as being unfamiliar to students. Twenty of 35 students in the first semester of the fifth grade who were asked to give the pronunciation and the meaning

for the name of the distributive principle gave the correct pronunciation while only one gave the correct meaning. The present investigation will provide additional information concerning students' recognition and use of the name applied to the distributive principle.

TABLE 1

Generic Types of Open Sentences[a]

Operation	Position of placeholder	Symmetric property of equality	
		a o b = c	c = a o b
Addition	c	a + b = □	□ = a + b
	b	a + □ = c	c = a + □
	a	□ + b = c	c = □ + b
Subtraction	c	a - b = □	□ = a - b
	b	a - □ = c	c = a - □
	a	□ - b = c	c = □ - b

[a]Table 1 is an adaptation from Table 1, Weaver, 1973a.
Small letters represent whole numbers.

Weaver's and Grouws' Studies

Weaver (1971, 1973a) focused on the ability to first-, second-, and third-grade students in 23 elementary schools within a city system to solve a number of different open sentence types involving addition and subtraction (approximately 1100 pupils per grade level participated in the investigation). Weaver identified twelve open sentence types based on the operations of addition and subtraction, the position of a placeholder, and the symmetric property of equality (see Table 1) and found differential achievement effects associated with these factors.

Grouws (1971, 1972) investigated the relationship of 32 third-grade students' performance in solving certain open sentences and the following three factors:

1. Open Sentence Type: ($N + \underline{a} = \underline{b}$; $\underline{a} + N = \underline{b}$; $\underline{a} - N = \underline{b}$; or $N - \underline{a} = \underline{b}$ where N is a placeholder for a whole number and $\underline{a}$ and $\underline{b}$ are whole numbers),
2. Context: (The open sentence presented with or without an analogous word problem), and
3. Number Size: (Large or Small).

Grouws reported the presentation of an analogous word problem along with an open sentence was not significantly more facilitative than the presentation of the open sentence alone. There were significant differences in students' solving performance on the four types of open sentences and the two levels of the number size factor. Grouws suggested that his investigation needed to be extended to other open sentence types involving other operations and other number domains. Since the introduction of the distributive principle at the elementary school level is closely associated with the use of open distributive sentences, certain factors associated with open sentences studied by Weaver and Grouws and/or other factors associated explicitly with open distributive sentences could affect students' performance when solving the open sentences. The present investigation will provide information concerning this effect.

Weaver's Study (1973c)

Weaver conducted a large scale study (1973c) using approximately 2000 students (Grades 4-7). An objective was to ascertain whether there existed differential achievement effects associated with factors and levels

identified in Table 2 as they relate to two variations of distributivity. These variations may be symbolized as

$$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (\underline{a} \times \underline{d})$$

and

$$(\underline{b} \times \underline{a}) + (\underline{c} \times \underline{a}) = (\underline{d} \times \underline{a})$$

where $\underline{a}$, $\underline{b}$, $\underline{c}$, and $\underline{d}$ are counting numbers such that $\underline{d} = \underline{b} + \underline{c}$. Weaver appropriately translated these variations into words or symbols indicated by levels of Factor A (Context), Table 2.

Two school districts participated in Weaver's investigation. The percent of correct responses[2] across items and tests presented to one school district was 2.5, 10.4, 10.9, and 14.7 for grades 4, 5, 6, and 7, respectively. For the other school district, the respective percents were 2.1, 6.3, 8.4, and 17.0. Weaver's study established that most elementary school students (Grades 4-7) have difficulty in finding a correct solution to the open sentences selected for use in his study.

A general lack of students' use of the distributive principle in attempting to solve the open distributive sentences was evident and among Weaver's conclusions and conjectures (1973b) was the following:

> As measured by the ability to give criterion responses [responses representing possible use of the distributive principle] to examples of the kinds tested, pupils' sensitivity to the use of distributivity was relatively low [p. 4].

Weaver also noted that his investigation did not enable one to infer whether students' relative low sensitivity to the use of distributivity

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[2] A correct response does not imply that the distributive principle was used. At least one other correct solution method exists (e.g., $(3 \times 4) + (3 \times 6) = (5 \times 6)$ when the sum of the indicated products is factored).

TABLE 2

Factors and Levels Investigated[a]

Factor	Level
A. Context	1. Regrouping sets 2. Multiplication-addition
B. Distributive form	1. Left (a x b) + (a x c) = (__x__) 2. Right (b x a) + (c x a) = (__x__) 3. Ambiguous (i x j) + (i x j) = (__x__)
C. Item-stem format	1. Horizontal example: (a x b) + (a x c) = (__x__) 2. Vertical example: (a x b) +(a x c) __x__
D. Item-response format	1. Left-open 2. Right-open 3. Wholly open
E. Size of common number	1. $4 < a < 10$; $1 < b < 10$; $1 < c < 10$, $b + c > 10$ 2. $11 < a < 17$; $1 < b < 10$; $1 < c < 10$; $b + c > 10$ 3. $4 < i < 10$; $11 < j < 17$; or $11 < i < 17$; $4 < j < 10$
F. School grade	1. Beginning 4th 2. Beginning 5th 3. Beginning 6th 4. Beginning 7th

[a] Table 2 is adapted from Table 1, Weaver, 1973b. Some details are omitted and others added. Weaver's Table should be consulted for further study of the factors and levels presented in his study.

was due to pupils' simply not "knowing" the distributive idea or to their failure to recognize its application in the situations presented. Thus, Weaver identified two plausible reasons for students' relative low sensitivity to use of distributivity in his study. Weaver (1973b) commented that

> . . . little if any systematic consideration appears to have been given to the possibility of differential effects associated with some . . . factors [See Table 2] and their potential interactions. This is so both for textbook material that has been prepared for elementary school pupils and for the limited empirical research that has been reported . . . . It very well may be that some of elementary school pupils' alleged difficulty with "the distributivity idea" is to be found in failure of mathematics programs to take into account conceptual differences inherent in certain of the factors and levels examined in the present investigation [p. 1].

The results of Weaver's investigation indicated a need to further study the relationship between elementary school students' performance in solving open distributive sentences and factors which could affect students' performance. Also, students' poor performance indicated a need to study students' solution procedures and errors made by students when solving open distributive sentences.

The Specific Problems to be Investigated

The research reviewed indicates a need for investigating the following two questions.

1. What is the relationship between students' use of the distributive principle as a method for solving open distributive sentences and selected levels of students' recognition of the distributive principle?

2. What is the relationship between students' use of the distributive principle or performance in solving open distributive sentences and selected factors associated with the open sentences?

The inability of Weaver's investigation to enable one to determine the relationship between students' use of the distributive principle as a method for solving open distributive sentences and selected levels of students' recognition of the distributive principle indicated a need to investigate Question 1. The observation that prior to Weaver's investigation little or no systematic consideration had been given to the relationship between students' performance in solving open distributive sentences and selected factors associated with the presentation of these open distributive sentences indicated a need to investigate Question 2.

In answering Question 1, levels of students' recognition of the distributive principle were determined by students' performance on a recognition level test. The Recognition Level Test and the selection of recognition levels are described in Chapter III. To answer Question 2, three factors involved in the presentation of open sentences were studied. These three factors (Open Sentence Type, Context, Number Size) and levels are described in the following discussion.

Open Sentence Type. The type of open sentence refers to one of the following two levels:

1. L-H denoting Left-Horizontal,
2. R-H denoting Right-Horizontal.

An example of each open sentence type is given below.

1. L-H: $(2 \times 3) + (2 \times 4) = (__ \times __)$.
2. R-H: $(3 \times 2) + (4 \times 2) = (__ \times __)$.

A need for investigating the effect of these two open sentence type on student performance is discussed in the following paragraph.

Schell's (1964) investigation included the teaching of the distributivity principle to each of two groups of students. One group of students was introduced to multiplication through the use of arrays; the second group was introduced to multiplication through a variety approach. Although difference between student performance on right- and left-horizontal open distributive sentences was not discussed in the reported study, students in one group were taught the distributive principle using only right-horizontal sentences; students in the second group were taught the distributive principle using only left-horizontal sentences. A different form of a 41-item inventory with each form containing 14 open distributive sentences[3] was administered to each group. All 14 items on the inventory administered to the first group were right-horizontal open distributive sentences; all 14 items on the inventory administered to the second group were left-horizontal open distributive sentences. The possible confounding in performance between the two treatment groups due to positioning of the common factor (i.e., left or right) in open distributive sentences was not discussed by Schell. Schell's (1964) conclusion was "There was no significant difference between the two groups on mean scores for the distributive property exercises, . . . [p. 92]."

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[3]The open distributive sentences used by Schell as items on his inventory and in the lessons presented to the two groups were not exactly the same as those used in the present investigation (e.g., Schell's tests included $(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = \underline{a} \times (\underline{b} + \underline{c})$ where $\underline{a}$, $\underline{b}$, and $\underline{c}$ represent whole numbers).

Schell's finding may be restated as follows: No significant differences were found between the two groups on mean scores for the distributive principle items when one group was taught the distributive principle using right-horizontal open distributive sentences only and the other group was taught the distributive principle using left-horizontal open distributive sentences only when each group was tested over the open distributive sentence type taught to that particular group. Thus, Schell's study provided no information concerning individual student's performance on both left- and right-horizontal open distributive sentences.

Weaver included left-horizontal and right-horizontal open distributive sentences in his investigation (1973c). Weaver was careful to emphasize that observations derived from his data are at most conjectures due to a lack of testing effects for statistical significance. However, two conjectures which seemed tenable from the collected data when open distributive sentences were presented in the multiplication-addition context were:

1. There would seem to be no difference in student performance when solving open distributive sentences due to the location of the common factor (i.e., right or left distributive form) and

2. There would seem to be no difference in student performance when solving open distributive sentences due to format (i.e., horizontal or vertical).

Although further study is needed to investigate the validity of both conjectures above, a need to limit the scope of the present investigation and the following considerations led the author to select left-horizontal and right-horizontal open sentence types for use in the present investigation.

1. Horizontal format was selected rather than vertical format since the horizontal format was considered by the author to be more adaptable when studying certain context factor levels.

2. Student performance on left-horizontal and right-horizontal open sentence types have not been explicitly investigated in a major study prior to Weaver's study (1973c). Schell did not explicitly study differential student performance due to these open sentence types. Further study was needed.

3. A possible conceptual difference associated with the selected open sentence types and certain context levels need investigation. This possible conceptual difference is explained in the following discussion of context levels selected for investigation.

Context. Four levels of context were used in presentation of the open sentence types. These four levels were designed to allow investigation of students' performance in recognizing the possibility of using the distributive principle when solving the open distributive sentences. These levels are defined as follow.

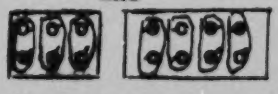
1. S: Each open sentence is presented symbolically as presented by Weaver (1973c). An example of an open sentence presented in the S context is

   $(2 \times 3) + (2 \times 4) = (__ \times __)$.

2. SP: Each open sentence is presented symbolically in conjunction with a picture.[4] Examples of a L-H open sentence and a R-H open sentence presented in the SP context are

   $(2 \times 3) + (2 \times 4) = (__ \times __)$

   and



   $(3 \times 2) + (4 \times 2) = (__ \times __)$.

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[4]Pictures of sets were used since some contemporary elementary school mathematics texts (e.g., Houghton-Mifflin, 1970; Silver-Burdett, 1970) introduce the distributive principle in conjunction with a joining of sets analogue. An additional reason for using sets is included in the discussion. Tacks were used as elements of the sets due to familiarity of tacks to students. Note that real tacks were not used but pictures of tacks.

3. SV: Each open sentence is presented symbolically along with a verbal statement informing the student that the distributive principle may be used. This statement is not included in directions given to students when open sentences are presented in the S or SP context.

4. SPV: Each open sentence is presented symbolically in conjunction with a picture as presented in the SP context and the verbal statement given in the SV context.

A study of how facilitative these context levels are in promoting students' use of the distributive principle as reflected in students' performance on open distributive sentences was warranted due to at least four reasons.

1. It seems that an assumption made by many curriculum writers (e.g., Duncan, Capps, Dolciani, Quast, and Zweng; 1970) is that the presentation of sets or arrays in a pictorial representation similar to that indicated by the SP or SPV context level along with open distributive sentences is facilitative in promoting an understanding and recognition of possible use of the distributive principle when finding a solution for open distributive sentences. Hobbs (1973) found no supportive evidence for this assumption when using the SP context level. No other reported research in this particular area of concern was found by the author. This investigation provided additional evidence in this particular area of need.

2. Weaver (1973c) found an indication of conceptual differences when students completed sentences of the form

   a sets of b and a sets of c are ___ sets of ___

   and

   b sets of a and c sets of a are ___ sets of ___

   where a, b, and c were counting numbers. These sentences were not presented with an accompanying picture. The author's assumption is that if conceptual differences are associated with sentences of the form concerned with regrouping sets, this conceptual difference would carry over to the use of pictures when solving the open distributive sentence presented in the SP or SPV context level.

3. The CAA reported small scale evidence indicating that students have difficulty in providing the proper meaning for the name given to the distributive principle. The inclusion of the SV context level and the SPV context level provided evidence concerning how facilitative certain verbal directions including the name given to the distributive principle are in promoting students' use of the distributive principle when solving open distributive sentences.

4. The contexts selected for use in the present research were considered by the author to be of extreme importance. A major reason for this importance is that most contemporary elementary school mathematics programs present the distributive principle and distributive sentence using one or a combination of these three contexts. For example, a student may be told to complete the open distributive sentence $(3 \times 4) + (3 \times 5) = (__ \times __)$ by using the distributive principle. In order to do what has been asked of the student, the student must understand the term "distributive principle." Otherwise the student could not follow directions and know for sure that the directions were being followed.

Number Size. Two levels of number size (Large and Small) were selected for use in generating open distributive sentences. The selection of these two levels was based on the following considerations.

1. Grouws (1971) reported significant differences in students' solving performance on selected open addition and subtraction sentences due to number size. The use of the distributive principle to find a solution for open distributive sentences requires no computational skills beyond that of addition regardless of number size. Finding a solution to an open distributive sentence may be one area in elementary school mathematics where use of small numbers is no more facilitative than use of large numbers.

2. An assumption by the author is that use of large numbers in open distributive sentences is more facilitative than use of small numbers in promoting use of the distributive principle by students solving the open distributive sentences.

The selection of the two levels of number size permitted the investigation of the two ideas expressed above. The small and large number domains are discussed in Chapter III.

Questions. In the investigation of the relationship between students' performance when finding a solution for the open distributive sentences and three factors (Open Sentence Type, Context, Number Size) associated with these open sentences, and in identification and classification of the methods attempted by students when finding a solution and the errors made, the following questions were investigated.

1. Is there a significant difference between means of correct responses for each of the two open sentence types?
2. Is there a significant difference among means of correct responses for each of the four contexts?
3. Is there a significant difference between means of correct responses for each of the two number sizes?
4. Is there a significant interaction between open sentence type and context?
5. Is there a significant interaction between open sentence and number size?
6. Is there a significant interaction between number size and context?
7. Is there a significant interaction between open sentence type, context, and number size?

Descriptive statistics were used for reporting identification and classification of methods attempted and errors made by students.

Summary

A number of investigations (e.g., Crawford, 1964; Gray, 1965; Schell, 1964) reported events implying elementary school students may have difficulty in using and understanding the distributive principle. No effort was made by these studies to focus on factors associated with the possible difficulty. Other investigations (Grouws, 1971, 1972; Weaver, 1971, 1973a)

reported differential student performance related to certain open sentences and selected factors associated with the open sentences. Weaver's investigation (1973c) of differential student performance related to selected open distributive sentences and a variety of factors associated with the open distributive sentences seems to have combined the difficulty students have in understanding and using the distributive principle with an extension of the study of differential student performance related to certain open sentences and selected factors associated with the open sentences. The fourfold purpose (see Chapter I) of the present investigation extended research as shown in Figure 2 by seeking answers to the questions indicated in this chapter and by seeking additional information concerning methods used and errors made by students when solving selected open distributive sentences. The design of the study including instruments for collecting data to investigate the indicated questions and to achieve the fourfold purpose of this investigation is presented in Chapter III.

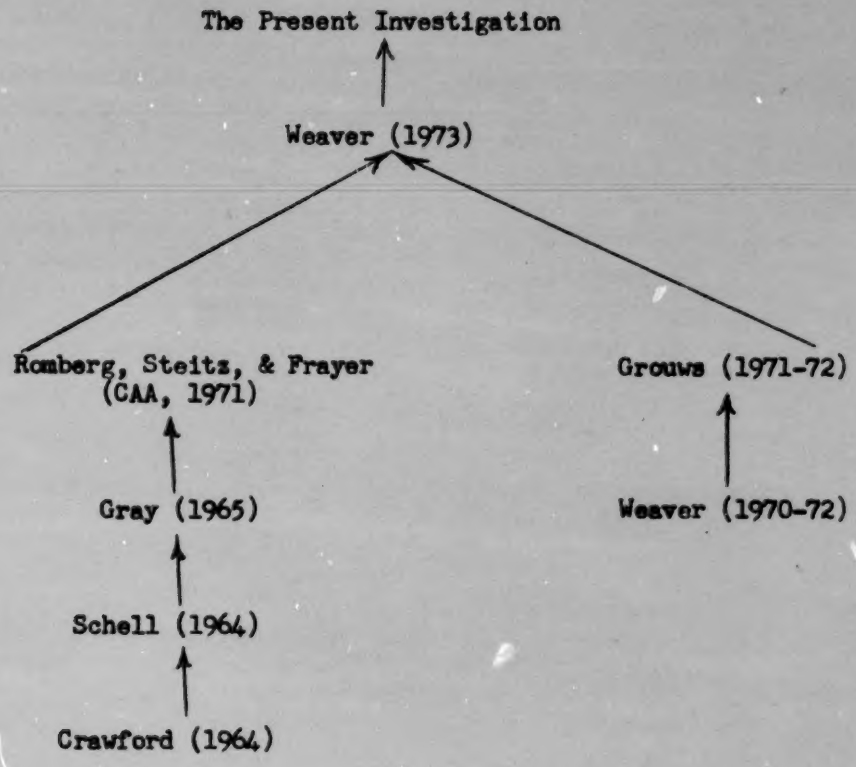

Fig. 2. The present investigation in perspective.

Chapter III

DESIGN OF STUDY

This chapter presents a brief overview of the study followed by a discussion of the student sample, the Recognition Level Test, the interview tests, the interviews, and the collection of data. The conduct of the study is presented in Chapter IV.

Brief Overview

The fourfold purpose of this investigation was

1. to investigate the relationship between elementary school students' performance when solving open distributive sentences of the form indicated in Chapter I and selected levels of three factors (Open Sentency Type, Context, Number Size) associated with these open sentences,

2. to investigate the relationship between elementary school students' use of the distributive principle as a method for solving the open distributive sentences and selected levels of students' recognition of the distributive principle,

3. to identify and classify other methods attempted by elementary school students when solving the open distributive sentences, and

4. to identify and classify errors in elementary school students' performance when solving the open distributive sentences.

Student performance on a recognition level test was used to assign each of 301 fifth-grade students to a level of recognition of the distributive principle. Four recognition level groups with each group of students reflecting a different level of recognition were selected for use in the

study.[1] Twenty students from each of the four groups were selected as subjects.

Each student was presented an interview test consisting of four open distributive sentences with each open sentence reflecting a different combination of the four possible combinations of the two levels of open sentence type and the two levels of number size. Each student's interview test was presented in one of the four context levels. The performance of each student on a four-item interview test and responses given during an individual interview provided data for use in achieving the fourfold purpose of the study.

The experimental design was a split plot design with repeated measures on the factor levels of open sentence type and number size. A representation of the design is given in Table 3.

Student Sample

The student sample was composed of fifth-grade students of middle class socio-economic status selected from three elementary schools located in the LaFollette Attendance Area of Madison (Wisconsin) Public Schools. Two reasons for selecting students from this attendance area are listed below.

1. The students' basic mathematics text introduced the distributive principle during the third grade through use of the S, SP, and SV context levels (Duncan et al., 1970). These three context levels were also used in the presentation of the distributive principle during the fourth grade with only the S and SV context levels used during the fifth grade. Since the relation between these context levels and student performance was investigated in the study, previous exposure of students to the variety of context levels was considered important.

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[1]The original design presented in this chapter contained only two recognition level groups. The change to four recognition level groups is explained in Chapter IV.

TABLE 3

Representation of Experimental Design

Recognition level groups	Context levels	L-H		R-H	
		L	T	L	T
1	S				
	SP				
	SV				
	SPV				
2	S				
	SP				
	SV				
	SPV				
3	S				
	SP				
	SV				
	SPV				
4	S				
	SP				
	SV				
	SPV				

Context
S....Symbolic
SP...Symbolic-Pictorial
SV...Symbolic-Verbal
SPV..Symbolic-Pictorial-Verbal

Number Size
L..Large
T..Small

Open Sentence Type
L-H..Left-Horizontal
R-H..Right-Horisontal

2. A portion of Weaver's data (1973c) was obtained from fifth-grade students attending other schools within the same attendance area. A comparison of results obtained from students within the same attendance area was made possible by selection of this sample.

A pilot study (Hobbs, 1973) and Weaver's study (1973c) indicated that students tend to find (or attempt to find) products while seeking a solution for open distributive sentences of the type being studied. A time element when interviewing made it desirable that students selected to participate in the study have experience in using the standard multiplication algorithm since numbers greater than ten were used in generating some open distributive sentences used in the study. Performance of students during the pilot study indicated that fourth grade was the highest grade level at which many students did not possess needed computational skill in using the algorithm. Checking multiplication skills was not an objective of the present investigation so the selection of subjects from a grade level higher than four was considered important, due to the time element involved in interviewing.

However, beyond fourth-grade level there is a general tendency by elementary school mathematics curricula to de-emphasize the use of sets in attempting to facilitate students' understanding and use of the distributive principle. This procedure was the one adopted by the basic text used by students participating in the study. Since the association of sets with open distributive sentences was involved in the study, it was considered important that students selected to participate in the study have recent exposure to the use of sets in association with open distributive sentences.

By selecting students from the fifth-grade level as subjects, the participants generally possessed the computational skill needed (i.e., most students could use the standard multiplication algorithm) and they had recently (within one year) been exposed to the SP context level in attempting to understand the use of the distributive principle. This latter characteristic was achieved by selecting only those students who had attended the selected schools for at least the past two academic years (1971-72 and 1972-73).

Recognition Level Test

The Concept Attainment Abilities Project (CAA) of the Wisconsin Research and Development Center developed tests using the Frayer, Fredrick, and Klausmeier (1969) schema for testing the level of concept mastery (Romberg, Steitz, & Frayer, 1971; Romberg & Steitz, 1971). The tests were developed by the CAA to check the attainment of selected mathematics concepts by intermediate grade students. A test to determine the assignment of students to selected levels of recognition of the distributive property was developed following procedures similar to those used by the CAA. This test is referred to as the Recognition Level Test in the following discussion.

Description of the Recognition Level Test. The Recognition Level Test contains 28 multiple choice items. Three of the items are used as sample items for introducing test procedures to subjects, 17 of the items are "dummy" items used to reduce learning effects which might be associated with the remaining 8 items. Student performance on the remaining eight items was used in determining a recognition level for each student. A

copy of the Recognition Level Test, a copy of the Recognition Level Test answer sheet, and a copy of Directions for Administration of the Recognition Level Test are found in Appendix A, Part I.

Generation of the Recognition Level Test. The first two sample items of the test were randomly selected from items on the 39 tests generated by the Concept Attainment Abilities Project. The third sample item was generated to serve an additional purpose as explained in the following discussion. The text series used by students participating in the study referred to the distributive principle as the "common factor rule" or "common factor property" in grade level 3. In grade level 4 and 5, two names (the distributive property and the common factor property) were applied. Due to students' exposure to both names, sample item 3 was included in an attempt to alert students to this terminology.

The 17 "dummy" items were also randomly selected from the 30 concept attainment tests generated by the CAA. The method used in selecting the 17 items was as follows. Seventeen four-digit numbers were randomly selected from a table of random numbers. The thousands and hundreds digits were used to designate a test number as sequenced in Working Paper 56 (Romberg and Steitz, 1971) and the tens and units digits were used to represent the test item which was selected from the test represented by the test number. For example, if 0205 was the four-digit number randomly selected then item 5 on the second test listed in Working Paper 56 was the item chosen for use. Noting that there are only 30 tests generated by the Concept Attainment Abilities Project and that each test contains 11 or 12 items, certain four-digit numbers were omitted. Also, no four-digit number was selected twice.

The paradigm used to generate the eight items was similar to that used by the Concept Attainment Abilities Project (Romberg, Steitz, & Frayer, 1971, p. 27) and is given in Table 4. The eight constructed items are given below.

1. Which equation uses the common factor property?

   A. $(4 \times 5) \times 6 = 4 \times (5 \times 6)$

   B. $(2 \times 8) = (8 \times 2)$

   C. $(7 \times 3) + (9 \times 3) = (7 + 9) \times 3$

2. Which equation does NOT use the common factor property?

   A. $(2 \times 5) + (7 \times 5) = (2 + 7) \times 5$

   B. $(3 \times 6) \times 4 = 3 \times (6 \times 4)$

   C. $(8 \times 9) + (8 \times 6) = 8 \times (9 + 6)$

3. The equation $(6 \times 4) + (5 \times 4) = (6 + 5) \times 4$ uses:

   A. the common factor property.

   B. the associative property.

   C. the commutative property.

4. The equation $(3 \times 5) + (3 \times 9) = 3 \times (5 + 9)$ does NOT use:

   A. the common factor property.

   B. the associative property.

   C. addends and factors.

5. Which of the following is always true about an equation using the common factor property?

   A. The equation uses a plus sign but not a multiplication sign.

   B. The equation uses a multiplication sign but not a plus sign.

   C. The equation uses both a multiplication sign and a plus sign.

TABLE 4

Paradigm Used to Generate Eight Items for Recognition Level Test

Number of item constructed	Given the...	the student can select the...
1	name "common factor property"	equation using the "common factor property"
2	name "common factor property"	equation not using the "common factor property"
3	equation using the "common factor property"	name of the property used
4.	equation using the "common factor property"	name of the property not used
5	name "common factor property"	statement always true about an equation using the "common factor property"
6	name "common factor property"	statement not always true about an equation using the "common factor property"
7	statement always true about an equation using the "common factor property"	name of the property the statement is true for
8	[a]name "common factor property"	equation using the "common factor property"

[a]The difference between the construction of item number 1 and item number 8 is that the equations used in item number 1 contain whole numbers and the equations used in item number 8 contain symbols representing numbers. Item number 8 is the only item containing equations not containing whole numbers.

6. Which of the following is NOT always true about an equation using the common factor property?

   A. The equation uses 4 as a common factor.

   B. The equation uses a plus sign and a multiplication sign.

   C. The equation uses parentheses.

7. Which of the following always uses a plus sign and a multiplication sign in the same equation?

   A. the commutative property

   B. the common factor property

   C. the associative property

8. An equation using the common factor property is:

   A. ☆ x (△ + □) = (☆ x △) + (☆ x □)

   B. ☆ + (△ x □) = (△ x □) + ☆

   C. ☆ x (△ x □) = (☆ x △) x □

Concerns in the item construction process were readability, validity, and reliability. The readability of the eight constructed items as well as the 17 "dummy" items and 3 sample items was established by a committee of three mathematics educators' professional judgment[2] and a small pilot study conducted by the author. Content validity was established by judgment of the author and the committee of three professional mathematics educators. The reliability study is reported in Chapter IV.

The 17 "dummy" items were assigned to item positions on the Recognition Level Test according to their order of selection with the exception that the eight items used for determining a recognition level for each

[2]The committee was three professors of mathematics education at the University of Wisconsin-Madison: Dr. J. Fred Weaver, Dr. M. Vere DeVault, and Dr. Thomas A. Romberg.

student were randomly assigned to item positions designated by numbers greater than three which are multiples of three. This procedure controlled for "distance" between the eight items. The assignment of the eight constructed items to item positions on the Recognition Level Test is given in Table 5 while the assignment of the 17 "dummy" items is given in Table 6.

Selected recognition levels. Criteria for different recognition levels were decided upon by the author and the committee of three mathematics educators. The eight items constructed for use in determining students' recognition levels were divided into two sets X and Y with four items in each set. Set X consists of items 12, 15, 18, and 21 on the Recognition Level Test while set Y consists of items 6, 9, 24, and 27. Set X contains items directly associated with a distributive sentence using whole numbers while set Y contains items not directly associated with a distributive sentence using whole numbers. The criteria for four recognition levels are summarized in Table 7. For example, recognition level I (RL-I) would be the level attained by a student correctly answering 3 or 4 items in set X and 3 or 4 items in set Y. Students to participate in the study were to be selected from RL-I and RL-IV.

Interview Tests

Each student selected to participate in the study was interviewed individually by the author. During the interview, students were administered an interview test in combination with one of the four context levels.

TABLE 5

Assignment of Eight Items
to Recognition Level Test

Number of item constructed	Position item was assigned to
1	15
2	12
3	18
4	21
5	6
6	9
7	27
8	24

Each student was administered only one interview test presented in only one of the context levels.

Description of interview tests. Each interview test consisted of four open distributive sentences with each sentence reflecting a different combination of the four possible combinations of the two levels of open sentence type and the two levels of the number size factors (i.e., R-H & L; R-H & T; L-H & L; L-H & T). A different item was placed on each of four 8 1/2" x 11" sheets of plain paper for presentation to students. An example of Interview Test 5 as printed for

TABLE 6

Assignment of "Dummy" Items
to Recognition Level Test

Four-digit numbers representing items	Position item was assigned to
0205	4
1110	5
2011	7
1509	8
2707	10
2210	11
1311	13
2510	14
211f[a]	16
0307	17
1302	19
2501	20
0408	22
2205	23
0501	25
1502	26
2605	28

[a]No item existed in item position 11 on test 21 generated by the Concept Attainment Abilities Project so the four-digit number 0411 was randomly selected to replace the four-digit number 2111.

TABLE 7

Criteria for Recognition Levels

Number of correct responses for items in set X	Number of correct responses for items in set Y	
	0, 1, or 2	3 or 4
3 or 4	RL-II	RL-I
0, 1, or 2	RL-IV	RL-III

RL---recognition level

presentation to students in combination with each of the four context levels is presented in Appendix A, Part II.

Generation of interview tests.[3] The procedure used in generating the five four-item interview tests used in the study is outlined below. Subsequent discussion adds more detail to steps IV-VI.

Step I. The set of four 4 x 4 Latin squares was generated.

Step II. Two of these standard Latin squares were randomly selected with the sequence of selection being noted for later use in Step V.

Step III. The selected standard Latin squares were randomized by randomizing rows and columns using random sequences (Winer, 1971).

Step IV. The resulting Latin Squares were used to generate the sequences for four items on each of the eight tests (four tests per Latin square).[4]

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[3]Originally more than 5 four-item interview tests were generated. Due to events reported in Chapter IV, only 5 interview tests were used. Consequently, only the generation of the 5 interview tests used in the study is described.

[4]Eight tests were generated and the five used in the study selected from those eight.

Step V. The 4 four-item tests resulting from use of the first selected standard Latin square were used as interview tests along with one of the 4 four-item tests resulting from use of the second selected standard Latin square.

Step VI. One large number triple and one small number triple were independently and randomly assigned without replacement from respective number domains (large and small) to each item position 1-4 on the interview tests.

The Latin squares resulting from Steps I-III and used in Step IV are presented in Figure 3 and denoted as A and B. Elements in rows of Latin square A were used to sequence the four items contained on Interview Tests I-IV and elements in rows of Latin square B were used to sequence the four items on Interview Tests V-VIII. Each of the four items reflecting different combinations of open sentence type and number size was assigned an item letter code as indicated in Table 8 to prevent confusion of item numbers with item position numbers. Table 9 contains the item sequences for the eight generated tests. The procedure for obtaining the item sequence is illustrated in the following discussion. The first row of

A	B
2 1 3 4	1 4 2 3
1 2 4 3	3 2 1 4
3 4 1 2	4 1 3 2
4 3 2 1	2 3 4 1

Fig. 3. Latin squares used to generate sequences for items on interview tests.

TABLE 8

Coding and Description of
Four Interview Test Items

Test item number	Item letter code	Item 2-level description
1	A	L-H; T
2	B	L-H; L
3	C	R-H; T
4	D	R-H; L

TABLE 9

Item Sequences for Eight Tests

Test No.	Item sequence number			
	1	2	3	4
I	B	A	C	D
II	A	B	D	C
III	C	D	A	B
IV	D	C	B	A
V	A	D	B	C
VI	C	B	A	D
VII	D	A	C	B
VIII	B	C	D	A

Latin square A was used to sequence the four items on Interview Test I. The second entry on the first row designates the use of item number 1 (Letter Code: A) as the second item on Interview Test I. This procedure was used in generating item sequences for each interview test. Tests I-IV were used as four of the five interview tests with Test VII being randomly selected from Tests V-VIII as the fifth interview test. The interview tests used in the study were renumbered as interview tests 1, 2, 3, 4, and 5 to prevent confusion in later discussion.

Two distinct number domains representing small and large number sizes were chosen for use in the study. The domains were denoted Small Number Domain and Large Number Domain reflecting the relative number size in each domain. The Small Number Domain consists of eight ordered number triples constructed from elements of the following sets using methods described below.

Set A = [$\underline{a}$ / $\underline{a}$ = 2, 3, 4, 5, 6, 7, 8, 9].

Set B = [($\underline{b}$,$\underline{c}$) / $\underline{c}$ + $\underline{b}$ > 10, $\underline{b}$ $\in$ A, $\underline{c}$ $\in$ A, $\underline{b}$ < $\underline{c}$].

Set C = [($\underline{c}$,$\underline{b}$) / ($\underline{b}$,$\underline{c}$) $\in$ B].

Ordered number triples ($\underline{r}$; $\underline{s}$,$\underline{t}$) were formed from the above sets using the following procedure [A similar procedure was used by Weaver (1973c).]. The elements for $\underline{r}$ were sequenced randomly without replacement from A. Four elements were randomly chosen without replacement from each of sets B and C. Each ordered pair selected from sets B and C was paired with an element chosen from A with the pairing based on the sequence of selection [e.g., If 4 was chosen as the second element for $\underline{r}$ and (9,2) was the second pair chosen for $\underline{s}$, $\underline{t}$, then the resulting ordered number

triple would be (4; 9,2).]. Ordered triples were selected such that each ordered triple has $\underline{r} \neq \underline{s}$, $\underline{r} \neq \underline{t}$, $\underline{s} \neq \underline{t}$, and instances where "commuted" pairs occur [e.g., $(\underline{t},\underline{s})$ and $(\underline{s},\underline{t})$] were avoided. These constraints were considered to be helpful when ascertaining methods used by students when solving open distributive sentences using the number triples selected from the large and small number domains.

The same procedure used in generating the eight ordered number triples in the Small Number Domain was used in generating the eight ordered number triples which are elements of the Large Number Domain. The three sets used in generating ordered triples for the Large Number Domain are described below.

Set $A' = [a' \;/\; \underline{a}' = \underline{a} + 10, \underline{a} \in \underline{A}]$.

Set $B' = [(\underline{b}',\underline{c}') \;/\; \underline{b}' = \underline{b} + 10, \underline{c}' = \underline{c} + 10, (\underline{b},\underline{c}) \in \underline{B}]$.

Set $C' = [(\underline{c}',\underline{b}') \;/\; (\underline{b}',\underline{c}') \in \underline{B}']$.

The resulting domains are given in Table 10.

From the eight elements in the Large Number Domain and from the eight elements in the Small Number Domain, one large number and one small number triple were independently and randomly assigned without replacement to each item sequence number 1-4 on interview tests as indicated in Table 11. This assignment was used in determining number triples used in the open distributive sentences on each interview test. For example, item sequence number 3 on Interview Test VII had the two-level description R-H; T (Tables 8 and 9). The open distributive sentence used as item 3 on Interview Test VII is

$$(4 \times 5) + (7 \times 5) = (__ \times __).$$

TABLE 10

Small and Large Number Domains

Small number domain	Large number domain
(3; 7,6)*	(16; 19,17)*
(9; 8,3)*	(17; 19,14)
(6; 8,5)	(14; 16,15)*
(8; 9,4)	(15; 17,14)
(7; 2,9)	(13; 15,19)
(4; 5,6)*	(19; 14,18)*
(2; 4,8)	(12; 16,17)
(5; 4,7)*	(18; 12,19)*

*Number triples randomly selected for use in generation of open distributive sentences for interview tests.

TABLE 11

Assignment of Number Triples
to Item Sequence Numbers

Item sequence number	Small number triple	Large number triple
1	(9; 8,3)	(18; 12,19)
2	(3; 7,6)	(16; 19,17)
3	(5; 4,7)	(19; 14,18)
4	(4; 5,6)	(14; 16,15)

The five interview tests which were used in the study are presented in Appendix A, Part II.

Assignment of students to interview test-context combinations. Each of the five interview tests was presented in each of the four context levels. These 20 combinations are referred to as interview test-context combinations throughout the following discussion.

An equal number of students was randomly chosen from students assigned to each of the recognition level groups selected for use in the study. The students selected from each group were assigned to interview test-context combinations such that one-fourth of the students selected from each recognition level group was assigned to interview tests presented in the S context level. An equal number of students from each recognition level group was assigned to interview tests presented in each of the three remaining context levels.

The Interviews

Individual interviews were used for collecting data. Each interview consisted of one session per student. Each student was presented a "warm-up" exercise followed by the presentation of an interview test in one of the context levels.

The "warm-up" exercise. A primary concern when interviewing a student is the student's response or lack of response to the interviewer's directions, comments, and/or questions. To provide each student an opportunity to adjust to the interviewer and interview situation, a "warm-up" exercise was designed. The primary objective of the "warm-up" exercise was to provide the interviewer an opportunity to establish the proper rapport with each student before proceeding with the presentation of an interview test within a given context. The "warm-up" exercise sheet is presented in Figure 4. A small cardboard triangular region (constructed to exactly cover the triangular region on the "warm-up" exercise sheet) was used in a discussion involving "slides, flips, and turns." No effort was made to maintain proper technical use of terms as this was not the objective of the "warm-up" exercise. The following sample discussion indicates the general procedure followed by the interviewer in presenting the "warm-up" exercise to each student. "I" represents the interviewer and "S" represents the student.

I: Here I have a sheet of paper (indicating "warm-up" exercise sheet). What do you see on this sheet of paper?

S: Right angles and triangles.

I. All right, each triangle has a right angle in it. Do you see anything else interesting about these triangles?

S: They're turned different ways.

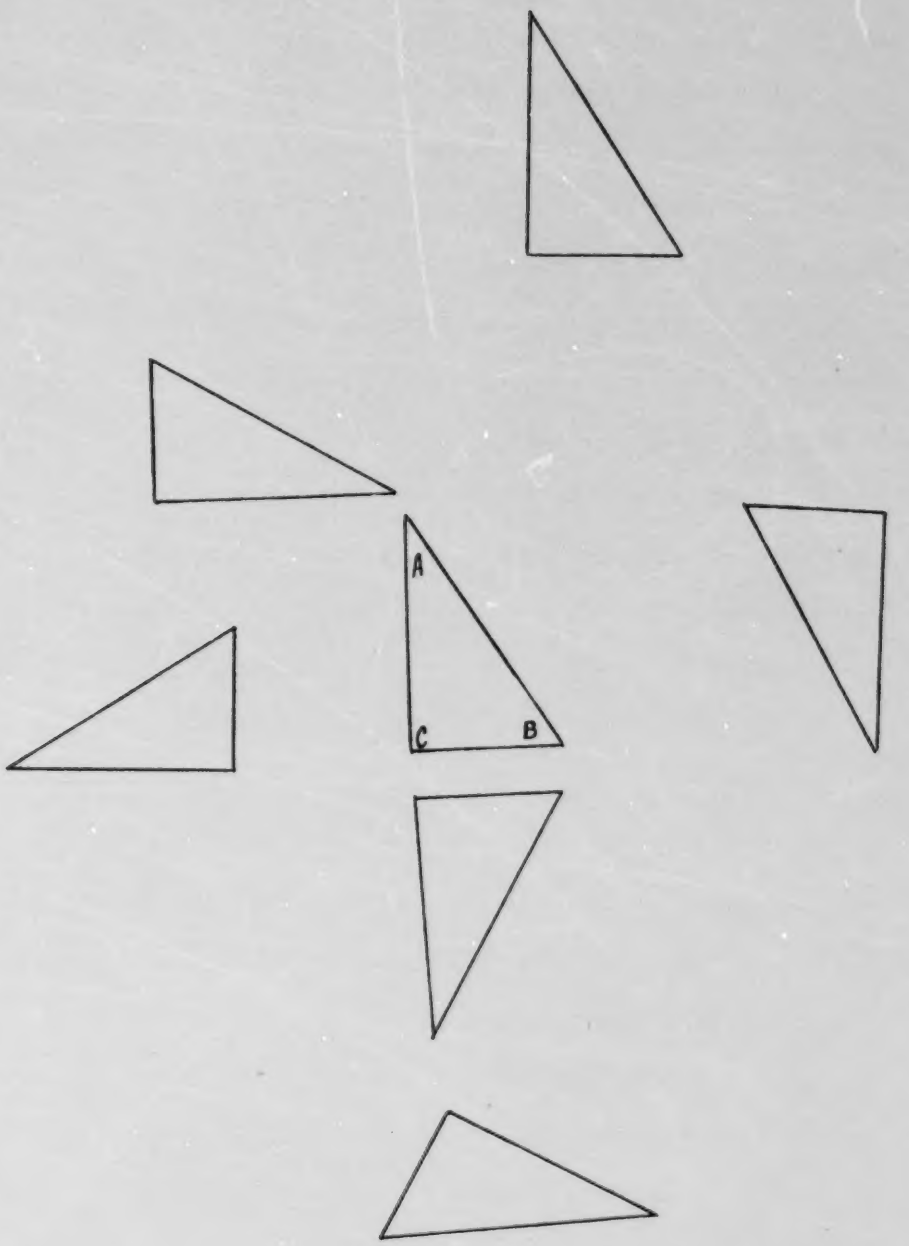

Fig. 4. "Warm-up" exercise sheet.

I: They are turned different ways. Anything else?

S: They have vertices.

I: They do have vertices. I've fixed this cover (showing small cardboard triangular region) for the triangles and I drew each triangle the same size so that this cover will cover each one. If I want to, I can move this cover by sliding it and it will exactly fit on there (illustrating movement of cover from one triangular region to another). Notice that all I had to do was slide it to cover that triangle (indicating triangular region which had been covered). Now I want you to put it over there (indicating another triangular region) and tell me what you do different from what I did.

S: Turn it over (student responds before moving the cover).

I: You want to try it?

S: (Student moves cover to indicated triangular region.) I didn't have to turn it over.

I: You didn't have to turn it over. What did you have to do?

S: Slide it. Slide it and turn it around.

(A procedure similar to the above was used with each student to establish proper rapport with students before presentation of interview tests.)

Administration of interview tests. The four items on a given interview test were presented one at a time for the student to solve. The following steps were followed in presenting each of the four items.

Step I: The open sentence was indicated by the interviewer as the comment, "Would you read this for me please?" was made.

Step II: If the student responded by reading the open sentence incorrectly, the interviewer commented, "Would you read this carefully once more please?" If the student had difficulty reading the open sentence, the interviewer gave help in reading the sentence.

Step III: After the student had correctly read the open sentence, one of the following sets of directions was given depending on the context being presented to the student.

S context: "Complete this (indicating the open sentence) by deciding what numbers should be placed on these blanks. You may figure on the sheet of paper. Talk out loud so I may understand how you decide. You may go ahead."

SP context: "Here are two cards (indicating the two cards). Each card contains sets of tacks. This card (indicating the left card) contains ___ sets of tacks with ___ tacks in each set. This card (indicating the right card) contains ___ sets of tacks with ___ tacks in each set. Complete this (indicating the open sentence) by deciding what numbers should be placed on these blanks. You may figure on the sheet of paper. You may use the sets to help you. Talk out loud so I may understand how you decide. You may go ahead."

SV context: "Complete this (indicating the open sentence) by deciding what numbers should be placed on these blanks. You may figure on the sheet of paper. You may use the distributive property or common factor property[5] to help you. Talk out loud so I may understand how you decide. You may go ahead."

SPV context: "Here are two cards (indicating the two cards). Each card contains sets of tacks. This card (indicating the left card) contains ___ sets of tacks with ___ tacks in each set. This card (indicating the right card) contains ___ sets of tacks with ___ tacks in each set. Complete this (indicating the open sentence) by deciding what numbers should be placed on these blanks. You may figure on the sheet of paper. You may use the sets to help you. You may use the distributive property or common factor property to help you. Talk out loud so I may understand how you decide. You may go ahead."

Step IV: The interviewer reminded the student to "talk out loud" no more than two times during the completion of a given open sentence.

Step V: After the student had selected numbers as a solution pair for the item, the interviewer asked, "Would you explain to me how you got the numbers you placed in the blanks so I may understand how you decided?"

Step VI: Clarification questions were asked if needed and if they were not considered confounding to the objectives of the study (i.e., with other items to be presented). Most clarification questions were asked following the student's completion of the four-item interview test.

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[5]The mathematics text used by the students participating in the study used "distributive property" and "common factor property" when discussing the distributive principle.

Interview Coding Form

In addition to the open distributive sentences being solved and the data generated from the students' correct or incorrect responses to these items, an attempt was made to ascertain methods used by students when solving the open sentences, the errors made, the use of the given context, the reading difficulty of the open distributive sentences, and students' reactions in general. Data were collected using audio-taping, possible writing (figuring by students), and numerical answers given by the students in responding to the open sentences.

An Interview Coding Form to be used when coding the data collected during the interviews was devised based on pilot study data (Hobbs, 1973), other research (Gray, 1965; Kilpatrick, 1967; Weaver, 1973c), and anticipated responses. A copy of the Interview Coding Form and a copy of Directions for Using Interview Coding Form are located in Appendix A, Part III. The reliability of the coding of interview data was based on agreement by the author and a graduate student in mathematics education. The result of the reliability study is presented in Chapter V.

Opportunity to Learn Survey

An Opportunity to Learn Survey form was designed to sample previous exposure of students at participating schools to various elements being studied during the investigation. This survey form was designed to be completed by teachers of the students participating in the study. A copy of the survey form is located in Appendix A, Part IV.

Summary

This chapter has presented a brief overview of the design of the study followed by a more detailed discussion of instruments used during the study, procedures used in selecting students to participate in the study, and procedures for collecting data to achieve the fourfold purpose of the study. While this chapter presents the original design of the study, certain modifications were made as a result of events occurring during the process of the study. These modifications and events are discussed in the following chapter.

Chapter IV

CONDUCT OF STUDY

This chapter presents the procedure of the study including certain modifications of the design.

Administration of Recognition Level Test

The Recognition Level Test was administered on Wednesday, March 21, 1973, to 303 fifth-grade students in three schools with four fifth-grade classes from each of two schools and five from the third school. At each school, the test was introduced and administered following the directions outlined in Appendix A, Part I. All testing was done simultaneously for all classes at a given school. The mathematics coordinator for the public school system and a graduate student in mathematics education helped the author administer the test.

At each of two schools, the population of fifth-grade students was assembled in one area and the test administered to the large group. At the third school, the graduate student administered the test to one intact class. The four remaining classes were assembled into two groups consisting of two classes per group. The author administered the test to one of these two groups and the mathematics coordinator administered the test to the remaining group. Circulation of the three test administrators among the students and the presence of teachers during the testing decreased

the probability that much exchanging of answers by students was likely. No time limit was set for the testing as each student was given as much time as desired to complete the test. Testing time per administration was approximately twenty minutes.

Formation of Recognition Level Groups

The formation of four recognition level groups according to the criteria given in Chapter III (Table 7) resulted in the number of students per group indicated by Table 12. Recognition level group I (RL-I) contains 26 students and recognition level group IV (RL-IV) contains 176 students.

The design of the study included choosing an equal number of students from recognition level group I and recognition level group IV. Due to the small number of students assigned to recognition level group I, a maximum of 52 students could be selected following the procedure of selecting an equal number of students from each of the two designated groups. At most, this procedure would result in 13 students performing on open distributive sentences presented in each of the four context levels assuming as equal number of students per context level. This number of students could decrease to an even smaller number following acquisition of parental permission for students to participate in the study. This small number of students was not considered adequate sample size when considering possible analysis procedures. Therefore, the students were reassigned to recognition levels according to criteria given in Table 13. The criteria are interpreted in the same manner as the criteria given in Table 7. For example, recognition

TABLE 12

Number of Students per Recognition Level

Number of correct responses for items in set X	Number of correct responses for items in set Y		Total
	0, 1, or 2	3 or 4	
3 or 4	RL-II 51	RL-I 26	77
0, 1, or 2	RL-IV 176	RL-III 48	224
Total	227	74	301[a]

RL-I....recognition level I
RL-II...recognition level II
RL-III..recognition level III
RL-IV...recognition level IV

[a]Two students were discarded from the study due to their failure to correctly record answers on the Recognition Level Test.

level group 2 contains all students who gave correct responses to three or four items in set X and to exactly two items in set Y on the Recognition Level Test.

This reassignment resulted in the number of students per recognition level group as indicated by Table 14. The original recognition level groups II-IV were broken into recognition level groups with each new group containing a smaller number of students. The original recognition level group I remained the same.

Approximately an equal number of students was assigned to recognition level groups 1-4. Chi-square tests for goodness-of-fit (Appendix E)

TABLE 13

Criteria for Recognition Levels 1-9

Number of correct responses for items in set X	Number of correct responses for items in set Y		
	0 or 1	2	3 or 4
3 or 4	RL 5	RL 2	RL 1
2	RL 7	RL 4	RL 3
1 or 2	RL 9	RL 8	RL 6

RL---recognition level

TABLE 14

Number of Students per Recognition Level 1-9

Number of students	Recognition level	Number of students	Recognition level
26	1	26	5
28	2	25	6
22	3	35	7
29	4	34	8
		76	9

indicate that the number of students assigned to each of the recognition level groups 1-3 is significantly different from the number which would result from guessing (alpha = .05) while the number of students assigned to recognition level group 4 is not. Following acquisition of parental permission for students to participate in the study, the number of students remaining in each of the recognition level groups 1-4 is found in Table 15. With approval of the committee of three mathematics educators, twenty students from each of these four recognition level groups were assigned to interview test-context level combinations. An explication of the selection of students is given in a following section.

TABLE 15

Number of Students in Recognition Levels 1-4
Following Parental Permission

Number of students	Recognition level
22	1
22	2
20	3
23	4

Selection of Interview Tests

From the set of interview tests originally generated for use with students, five tests were chosen for use with students selected from the four recognition level groups. The five interview tests were chosen as follows.

1. The first four interview tests originally generated were selected. Two rationale for this selection are 1) these tests were the first four interview tests generated for the original design and 2) the author wished to control to some extent the number size used in the first open sentence presented to students.

2. The fifth interview test was randomly selected from the second set of four interview tests generated in the original design.

Selection and Assignment of Students

Twenty different interview test-context level combinations resulted from the four context levels and five interview tests. Twenty students were selected from each of the four recognition level groups with the restriction that eight students of one sex and twelve students of the other sex were selected from each of the four groups. The number of students per sex was based on the number of each sex available within each group and the need to balance sex effects (if any existed) between context levels. If more than the desired number of students of a given sex per group was available, random selection of the number of students needed was used to determine which students of that sex were to participate in the investigation. Within each group, interview tests were randomly assigned to students with the restriction that if an interview test-context level combination was assigned to a student of a given sex, the remaining combinations of context levels with that particular interview test were

randomly assigned to three students of the same sex as the student first selected.

Five sets of four students were formed within each group of twenty students representing a particular recognition level. Each of the five sets contained four boys or four girls. Members of each set were determined by the random sequence resulting from selection of students to participate in the investigation. As an example, suppose eight boys and twelve girls assigned to recognition level 1 were selected to participate in the interviews with the random sequence of boys chosen represented by $b_1, b_2, \ldots, b_8$ and the random sequence of girls chosen represented by $g_1, g_2, \ldots, g_{12}$. The sets of four students b_1-b_4, b_5-b_8, g_1-g_4, g_5-g_8, g_9-g_{12} formed the five sets representing recognition level 1.

To determine if a set of boys (B) or a set of girls (G) should be assigned to a given interview test, the 5 x 5 matrix given in Figure 5 was generated. Sex represented by the first entry was determined randomly and an alternating procedure used to generate other entries for rows 1-4. The five interview tests (1-5) were randomly sequenced to form the fifth row. Each entry in the first row represents a set of four students to be assigned by indicated sex to recognition level 1, each entry in the second row represents a set of four students to be assigned by indicated sex to recognition level 2, etc. Sets of students within a given column were to be assigned to the interview test represented by the number found in that column. But since the actual number of available boys and girls in the recognition level groups did not permit a distribution in accord with Figure 5, the first three entries in the last column representing

B G B G B

G B G B G

B G B G B

G B G B G

5 1 4 2 3

Fig. 5. First matrix for assignment to interview tests.

sex were changed as noted in Figure 6 to fit the existing boy-girl distribution. The matrix given in Figure 6 was used for assignment of sets of students to interview tests. Context levels were randomly assigned within each set of students yielding a different context level for each student within a given set.

Each student within each recognition level group was assigned to the interview-context level combination indicated in Table 16. The

B G B G G

G B G B B

B G B G G

G B G B G

5 1 4 2 3

Fig. 6. Final matrix for assignment to interview tests.

TABLE 16

Students' Assignment by Group[a]
to
Interview Test-Context Level Combinations

Interview test	Context level	Groups 1	2	3	4
1	1	09**	27*	50**	74**
	2	10**	40*	44**	61*
	3	14**	38*	57**	78*
	4	08**	24*	55**	70*
2	1	19**	26*	45**	63*
	2	11**	29*	52**	69*
	3	07**	32*	60**	62*
	4	20**	39*	51**	73*
3	1	13**	34*	53**	76**
	2	12**	37*	48**	68**
	3	15**	31*	54**	64**
	4	18**	21*	46**	72**
4	1	04*	22**	43*	77**
	2	16*	36**	59*	66**
	3	17*	30**	56*	79**
	4	01*	25**	42*	71**
5	1	03*	28**	47*	80**
	2	02*	33**	41*	65**
	3	06*	35**	49*	67**
	4	05*	23**	58*	75**

*boys
**girls

[a] Students were assigned numbers such that students 01-20 represent recognition level 1, students 21-40 represent recognition level 2, students 41-60 represent recognition level 3, students 61-80 represent recognition level 4.

procedure used in the assignment resulted in eleven girls and nine boys assigned to each context level. Eight girls and twelve boys were selected from recognition level 2 while twelve girls and eight boys were selected from each of the remaining recognition level groups.

The Interviews

Information concerning each school's daily schedule for the fifth-grade classes, special programs scheduled, field trips, etc. was obtained and interviewing at schools scheduled to avoid conflicts. Interviewing was to be completed during the first three weeks of April, 1973 with the interviews being completed at a different school each week. Interviewing was scheduled for Tuesdays, Wednesdays, and Thursdays beginning at approximately 8:45 a.m. and not extending past 2:00 p.m. and for Friday mornings. The sequencing of student interviews was as follows:

1. Within each school, classes from which students were selected were randomly sequenced.[1]

2. Within each class, students to be interviewed were randomly sequenced.

This procedure was an attempt to randomize students' discussion following interviews among students assigned to different recognition levels and interview test-context combinations. No feedback concerning discussion of the type referred to was noticeable during conversations with students.

The interviews proceeded according to schedule except for the following four events.

1. Bad weather caused the closing of schools for one day.

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[1]Since two classes at one school held math class together, these two classes were considered as one in the random sequencing.

2. A "Spelling Bee" was rescheduled at one of the schools at a conflicting time.

3. During one interview, the author inadvertently skipped the presentation of the last of the four open sentences to be presented to the student.

4. One student was absent on the day when she was supposed to be interviewed.

The closing of schools one day and the rescheduling of the "Spelling Bee" caused a change in the completion of one school per week. As a result, interviewing at one school was split between two weeks. This did not seem to affect results as reflected by student performance or by conversation with the students regarding possible student feedback from one week to the other. The student who was absent and the student who had not performed on one of the four open sentences attended the same school, so a return to that school was scheduled to gather the missing data. Again, no feedback was evident. Thus the four events above did not seem to affect the performance of students.

A typical interview could be outlined as follows.

1. The student was met at the door of the classroom and escorted to the interview room. During the walk to the interview room, the author engaged the student in informative talking[2] as well as trivial conversation.

2. The student and author entered the interview room where two chairs and a table containing a tape recorder, a pencil, the "warm-up" exercise sheet, and the student's interview packet[3] had been placed.

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[2]"Informative talking" may be considered as talk used to gain information.

[3]An interview packet was an envelope containing the interview test to be presented to the student.

3. The interview began with the "warm-up" exercise, moved into the presentation of the interview test followed by more general questioning and exploration of the student's performance. This latter exploration involved student's performance on additionally selected open sentences and/or questioning based on the student's performance on the four open sentences presented on the interview test.

4. The student was escorted back to the classroom and another student obtained for the next interview. The time for an interview was approximately 30 minutes.

Opportunity to Learn Survey

An Opportunity to Learn Survey form as found in Appendix A, Part IV was given to each third-, fourth-, and fifth-grade teacher at each of the three schools. The main objective of this survey was to obtain additional information concerning students' opportunity to learn to recognize and use the distributive principle in solving open distributive sentences associated with the factors being investigated. The Recognition Level Test did not provide information concerning students' recognition of the distributive principle in relation to the factors associated with open distributive sentences presented in the investigation.

The distributing and collecting of the survey forms were completed during the last week of interviewing. Thirty-two of thirty-five forms distributed were completed and returned.

Gathering of Student Information

Schools supplied the following data on individual students for use in the study if needed.

1. IQ based on results from the Lorge-Thorndike IQ Test administered to the students during the second grade (1969-70).

2. Mathematics achievement scores based on results from the STEP Math Concepts Tests administered to the students during the fifth grade (1972-73).

3. The length of time in attendance at the particular school and school system.

Characteristics of Student Sample

Differences or lack of differences in performance between groups or within groups of students may be attributable to certain variables such as intelligence, mathematics achievement, etc. Data concerning certain of these variables are presented in this section.

Opportunity to Learn

The 80 students who participated in the investigation had attended respective schools in the Madison public school system for the academic years 1971-72 and 1972-73. The Opportunity to Learn Survey (see Appendix A, Part IV) was used to gain information from teachers with regard to students' exposure to the joining of sets, the joining of sets related to the distributive principle, the "idea" of distributivity, the term "common factor rule," the term "distributive property," the "use" of the distributive principle, equations like $(3 \times 4) + (3 \times 6) = (3 \times 10)$, and equations like $(3 \times 4) + (3 \times 6) = 3 \times (4 + 6)$.

Exposure of students to each of the eight mathematics ideas listed above was indicated by teachers on a continuum ranging through five categories. The five categories are listed below and were assigned numerical values 1-5 as indicated: poor (1), fair (2), moderate (3), quite good (4), very good (5). The means for each category per grade level are presented

in Table 17.[4] For example the exposure of students to the joining of sets related to the distributive principle at the fourth-grade level had a mean of 3.4. This implies that on the average, teachers considered students who attended fourth grade at the three schools had slightly more than moderate exposure to the relationship being surveyed. This indicates that students were exposed to the relationship between the joining of sets and the distributive principle at the fourth-grade level. The extent of this exposure at each school and within a particular class cannot be obtained from the data. The intent was to provide evidence that students had been provided exposure to certain mathematical ideas being considered in the investigation.

Although exposure to mathematics ideas and recognition of mathematical ideas provide some information about students' opportunity to learn, at least two other variables providing information about students' opportunity to learn are the content presented and the procedure used in presenting the content. Students in schools 1 and 2 were presented the content contained in the Houghton-Mifflin series (Duncan et al., 1970) at the fifth-grade level in a traditional manner (i.e., through presentation of material to the class as a unit . . . not individualized to any great extent). Students in school 3 were presented the content at the fifth-grade level through an individualized program using student packets. The individualized program was begun the academic year 1972-73 and consisted of 30 individual packets based on content in the Houghton-Mifflin series. Packets 1-4 were basic computational reviews covering third-grade level materials, packets 5-16

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[4]Teachers' responses were recorded on a continuum but the author assigned numerical values according to the closest of the five categories indicated on the continuum.

TABLE 17

Summary of Exposure of Students to Selected Mathematics Ideas

Grade level	Selected mathematics ideas							
	Joining of sets	Joining of sets related to the dis. principle	"Idea" of distributivity	Term "common factor rule"	Term "distributive property"	Use of dis. prin.	Equations[a] A	Equations[a] B
5	4.0	3.2	3.3	2.8	2.2	3.0	3.1	3.0
4	4.4	3.4	3.2	3.1	3.4	3.3	3.5	3.3
3	2.9	2.5	2.6	1.8	2.2	2.6	2.5	2.7
Average	3.8	3.0	3.0	2.6	2.6	3.0	3.0	3.0

[a]Equation A was an equation like $(3 \times 4) + (3 \times 5) = 3 \times (4 + 5)$ and equation B was an equation like $(3 \times 4) + (3 \times 5) = (3 \times 9)$.

covered fourth grade materials, packet 17 was a review packet, and packets 18-30 covered fifth-grade content.

Characteristics of Students

Table c of Appendix C contains information about selected characteristics of individual students. Table 18 summarizes selected characteristics of the student sample which are discussed in the following paragraph.

TABLE 18

Selected Characteristics of Students

Selected characteristic[a]					
Mean IQ	Mean mathematics achievement	Mean concept attainment	Number of students/school		
			School 1	School 2	School 3
109.3	27.3	10.2	29	17	34

[a]IQ, mathematics achievement, and concept attainment are based on administration of Lorge-Thorndike IQ Test (1969-70), STEP Math Concepts Test (Oct. 1972), and Recognition Level Test (1973), respectively.

IQ data were collected from the files of the three schools attended by students who participated in the investigation. The IQ of each student was based on an administration of the Lorge-Thorndike IQ Test during the academic year 1969-70. The mean IQ of students selected was 109.3. Mathematics achievement data were obtained from administration of the STEP Math Concepts Test, Series 2, Form 4-A in October, 1972. The mathematics achievement mean (based on a possible raw score of 50 per student) for the selected students was 27.3. The Recognition Level Test (see Chapter III) contained 17 "dummy" items which were used to tabulate a mathematics concept attainment score for each of the 80 students participating in

the investigation. The concept attainment mean for the selected students was 10.2. Twenty-nine students were selected from school 1, 17 from school 2, and 34 from school 3. The selected sample of students contained 36 boys and 44 girls.

Reliability of Recognition Level Test

Hoyt's coefficient of internal consistency was determined for each of the two sets of items contained on the Recognition Level Test (set X consisting of constructed items 1, 2, 3, and 4; set Y consisting of constructed items 5, 6, 7, and 8; see Chapter III). The coefficient of internal consistency for set X was 0.39 and the coefficient of internal consistency for set Y was 0.27. Although these coefficients may seem small, the importance of test length in determining the size of coefficients of reliability is illustrated by the Spearman-Brown formula. If the number of items on a test is increased (assuming items added are equal in quality to the original ones), the reliability of the test will always improve noticeably. Using Spearman-Brown's formula, the addition of 12 items to set X and the addition of 12 items to set Y would increase reliability coefficients to 0.72 and 0.60, respectively. The small number of items in set X and the small number of items in set Y should be considered in judging the reliability coefficients. There are also other factors which affect the magnitude of a reliability coefficient and these factors need considering. Among these are characteristics of the population tested, performance of the population, standardization of testing conditions, guessing by students, and a possible conflict between selecting items

for validity and selecting items for reliability. Consequences of the low reliability of the Recognition Level Test are considered in Chapter V.

Summary

This chapter has presented the conduct of the study including deviations from and/or modifications of the original design of the study which was presented in Chapter III. The data collected and used in analysis are presented in the following chapter.

Chapter V

RESULTS OF STUDY

This chapter presents results of the performance of the 80 students (20 students in each of four recognition level groups) on interview test-context combinations which were collected and used. Data and results of statistical tests used in analyzing data are presented according to eight broad categories: Coding of Interview Data, Solution Methods, General Performance, Errors, Relationship Between Performance and Factors, Relationship Between Performance and Recognition Level Groups, Characteristics of Students and Contexts, and Characteristics of Students Using the Distributive Principle.

Coding of Interview Data

During individual interviews, data were collected via audio-taping of verbal responses, writing (figuring) by students on interview tests, and recording of numerical solutions found for each open distributive sentence. After each student had been presented an interview test in a given context, collected data representing the student's performance were coded and recorded on the Interview Coding Form (The Interview Coding Form and Directions for Using Interview Coding Form are located in Appendix A, Part III). Components of student performance which were coded are found in the reliability study of the author's coding of interview data.

Directions for coding these components are found in Appendix A, Part III.

Reliability of Coding Interview Data

The reliability of the author's coding of interview data reflecting students' performance was collected after all 80 students had been interviewed and all collected data had been coded by the author. The procedure employed is outlined below.

Thirty-two students were selected from the 80 students interviewed. The method used in selecting the 32 students was random selection of two students per context per recognition level group. This selection provided eight students per context and eight students per recognition level group. For each student selected, one of the four different combinations of open sentence type and number size (i.e., left-horizontal and small, left-horizontal and large, right-horizontal and small, right-horizontal and large) was randomly selected to be used in the reliability study. The combination selected per student indicated the performance of each student which was used in the reliability study (i.e., interview data resulting from the student's performance when presented an open distributive sentence reflecting the selected combination was recoded during the reliability study). Table 19 presents the students selected, the context and group represented, and the open distributive sentence description selected per student.

A graduate student in mathematics education was given a copy of the Interview Coding Form and a copy of Directions for Using Interview Coding

TABLE 19

Identification of Interview Data Recoded
in
Reliability Study

Student	Recognition level group represented	Context student's interview test was presented in	Open distributive sentence selected[a]
09	1	S	1
19	1	S	2
16	1	SP	3
02	1	SP	1
20	1	SV	3
18	1	SV	4
07	1	SPV	2
15	1	SPV	1
27	2	S	1
22	2	S	2
37	2	SP	2
40	2	SP	1
24	2	SV	2
23	2	SV	1
30	2	SPV	1
38	2	SPV	3
53	3	S	1
45	3	S	1
59	3	SP	3
48	3	SP	3
46	3	SV	2
55	3	SV	3
57	3	SPV	4
54	3	SPV	4
77	4	S	4
80	4	S	3
61	4	SP	2
68	4	SP	4
70	4	SV	3
73	4	SV	3
62	4	SPV	1
64	4	SPV	3

[a] 1: left-horizontal; small number size
2: left-horizontal; large number size
3: right-horizontal; small number size
4: right-horizontal; large number size

S: symbolic
SP: symbolic & pictorial
SV: symbolic & verbal
SPV: sym., pict., & verbal

Form two weeks prior to the graduate student's coding of interview data. One day prior to the graduate student's coding of interview data, the author met with the graduate student. Data resulting from students' performance when presented approximately seven interview test items were coded independently by the author and the graduate student and then individual codings were compared. No student's performance data was coded at this session if that student's performance had been selected for recoding in the reliability study. Questions which the graduate student had with regard to the coding scheme were answered by the author. The session lasted approximately an hour.

The graduate student was given a tape recording of the interview data to be coded and the student interview packets which were needed. The graduate student independently coded performance of students on the randomly selected interview test items indicated in Table 19. A comparison of the graduate student's coding of the selected interview data with the author's coding of the same interview data showed the following agreement. The graduate student and the author were in perfect agreement (100%) on coding the 32 students' scores and on coding difficulty when reading the open distributive sentences. Agreement on coding of the solution method used, multiplication method used, addition method used, factoring method used, checking of solutions by students, students' use of context level, and students' errors was 87.5%, 90.6%, 90.6%, 90.6%, 96.9%, 96.9%, and 84.6%, respectively. More detailed results of the reliability study are presented in Appendix D.

General Performance

General performance was characterized by a set of two components of performance reflected by student responses during individual interviews. These two components were 1) number of correct solutions [CS] and 2) number of times the distributive principle was used as a solution method [DP]. In neither instance was there a significant difference between boys' and girls' performance levels, thus permitting a pooling of data across sexes.

Correct Solutions

Each of 80 students was presented an interview test containing four open distributive sentences. No student gave more than one solution per open sentence. Fifty-five correct solutions were given out of a total of 320 solutions (four open sentences/student x 80 students). Ten students found a correct solution for each of the four items, no student found a correct solution for exactly three items, five students found a correct solution for exactly two items, five students found a correct solution for exactly one item, and sixty students failed to find a correct solution for any item. The mean number of correct solutions per student was 0.69 and the variance was 1.86. Tables B-1 and B-2 in Appendix B contains data for each student's performance in obtaining correct solutions for items presented on interview tests.

Use of the Distributive Principle

The distributive principle was selected as a solution method 46 times out of a total of 320 (four open sentences/student x 80 students).

Ten students used the distributive principle when finding a solution for each item, one student used the distributive principle exactly three times, one student used the distributive principle exactly two times, one student used the distributive principle exactly one time, and 67 students failed to use the distributive principle. The mean number of times the distributive principle was used per student was 0.58 and the variance was 1.87.

Solution Methods

One purpose of the study was to identify and classify solution methods used by elementary school students when attempting to find solutions to open distributive sentences. The following discussion presents the identification and classification of solution methods used by students when attempting to solve open distributive sentences. Data related to student selection of various solution methods are presented in a following section. Solution methods used by students were determined independently of students' finding correct or incorrect solutions to given open distributive sentences.

Identification and Classification of Solution Methods

Eight solution methods[1] were identified and classified according to two categories: correct and incorrect. Two correct solution methods were identified and are described below.

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[1]The solution methods are the same as those identified by Weaver (1973c). Interviews provided an opportunity for greater accuracy in identifying solution procedures.

Method 1: Use of the distributive principle
[e.g., (3 x 9) + (3 x 5) = (3 x 14)]

Method 2: Factoring the sum of the two indicated products[2]
[e.g., (3 x 9) + (3 x 5) = (2 x 21)]

Six incorrect solution methods were identified and are described below.

Method 3: The two indicated products were used as a solution pair
[e.g., (3 x 9) + (3 x 5) = (27 x 15)]

Method 4: The sum of the common factor added to itself and the sum of the two "uncommon" factors were used as a solution pair
[e.g., (3 x 9) + (3 x 5) = (6 x 14)]

Method 5: The sum of the numbers in the first parenthesis and the sum of the numbers in the second parenthesis were used as a solution pair
[e.g., (3 x 9) + (3 x 5) = (12 x 8)]

Method 6: An attempt was made to solve the open distributive sentence but no definite procedure indicated.[3]

Method 7: No attempt was made to solve the open distributive sentence.

Method 8: A procedure not described by methods 1-7

Table 20 presents a summary of the identification and classification of solution methods along with a more generalized symbolic form and the identification code representing each solution method. The solution methods are referred to by code in the following discussion.

Errors

One purpose of the study was to identify and classify errors in elementary school students' performance when solving the open distributive

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[2]"Indicated products" are products indicated by parentheses in the open distributive sentence.

[3]Actually more than one solution procedure may be identified in the set of solution methods coded 6 or 8. These methods are discussed in a following section.

TABLE 20

Identification and Classification
of
Solution Methods

Classification	Identification code	Verbal and symbolic description[a]
Correct	1	$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (\underline{a} \times \underline{d})$ where $\underline{b} + \underline{c} = \underline{d}$ and the distributive principle was used
	2	$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (\underline{e} \times \underline{f})$ where $\underline{e}$ and $\underline{f}$ are determined by factoring the sum of the indicated products
Incorrect	3	$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (\underline{g} \times \underline{h})$ where $\underline{g} = \underline{a} \times \underline{b}$ and $\underline{h} = \underline{a} \times \underline{c}$
	4	$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (\underline{i} \times \underline{j})$ where $\underline{i} = \underline{a} + \underline{a}$ and $\underline{j} = \underline{b} + \underline{c}$
	5	$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (\underline{k} \times \underline{l})$ where $\underline{k} = \underline{a} + \underline{b}$ and $\underline{l} = \underline{a} + \underline{c}$
	6	An attempt was made to solve the open distributive sentence but no definite procedure indicated
	7	No attempt was made to solve the open distributive sentence
	8	A procedure not described by solution methods 1-7

[a]Small letters which are underlined represent whole numbers. Also, only symbolic descriptions using left-horizontal distributive sentences are presented. Analogous descriptions may be generated for right-horizontal distributive sentences.

sentences. Errors reflected by students' performance were identified and classified according to the following categories: Incorrect Use of the Distributive Principle, Computation Errors, Use of Incorrect Solution Methods, and Reading Difficulty (i.e., difficulty in reading open distributive sentences). Since errors made by students were dependent to a large extent on students' selection of solution methods, data reflecting students' selection of solution methods are presented prior to a discussion of errors within each category.

Use of Solution Methods

Data related to students' use of solution methods are summarized in Table 21 and discussed in the following paragraphs.

A correct solution method was selected a total of 76 times by the 80 students (a mean of 0.95 times per student) when presented 320 open distributive sentences (four per student). The percent of total times (% of 320) was 23.7 with 21 students selecting a correct solution method at least once. Solution method 1 (see Table 20 for description of solution methods) was selected a total of 46 times by the 80 students (a mean of 0.575 times per student). The percent of total times (% of 320) was 14.4 with 13 students selecting solution method 1 at least once. Data related to student selection of solution method 2 are found in Table 21.

An incorrect solution method was selected a total of 244 times by the 80 students (a mean of 3.05 times per student) when presented 320 open distributive sentences (four per student). The percent of total times (% of 320) was 76.3 with 63 students selecting an incorrect solution method at least one time. The most frequently selected incorrect

solution method (solution method 3) was selected a total of 198 times by the 80 students (a mean of 2.475 times per student). The percent of total times (% of 320) was 61.9 with 54 students selecting solution method 3 at least once. As indicated by data summarized in Table 21, selection of

TABLE 21

Summary of Solution Method Usage

Solution method	Number of students using method at least once	Total number of times used by 80 students	% of total number of times (% of 320)	Mean number of times per student (80 students)
		Correct methods		
1	13	46	14.4	0.575
2	8	30	9.3	0.375
Total	21[a]	76	23.7	0.950
		Incorrect methods		
3	54	198	61.9	2.475
4	4	8	2.5	0.100
5	2	4	1.3	0.050
6	5	10	3.1	0.125
7	0	0	0.0	0.000
8	9	24	7.5	0.300
Total	63[a]	244	76.3	3.050

[a]The number of students using a correct solution method at least once may not be the sum of the numbers of students using each correct solution method at least once. This statement is also true for the number of students using an incorrect solution method at least once.

other incorrect solution methods was minimal compared to use of solution method 3. The second and third most frequently used incorrect solution method codes were 8 and 6, respectively. Due to more than one procedure being identified in the set of procedures represented by each of these codes, solution procedures assigned codes 6 and 8 are discussed in the following paragraphs.

Procedures by five students were identified and coded as solution method 6. Eight of the ten times solution method 6 was assigned to students' procedures resulted when open distributive sentences containing large numbers were presented. Four of the five students' procedures were coded as solution method 6 when students could not "handle" multiplication of the large numbers (two-digit numbers). Performance by these four students accounted for six of the ten times solution method 6 was assigned. The other four times resulted from a fifth student's procedure when the student did not respond to any of the four items on an interview test.

Procedures by nine students were identified and coded as solution method 8. Table 22 summarizes the 24 solutions given by students with a procedure coded as solution method 8.

Incorrect Use of the Distributive Principle

The distributive principle was classified as being used incorrectly by three students a total of ten times. The same error was made by each of the three students with each student multiplying rather than adding the "uncommon" factors [e.g., $(3 \times 4) + (3 \times 5) = (\underline{3} \times \underline{20})$].

TABLE 22

Summary of Solutions Given by Procedures Identified as Solution Method 8

Solution given[a]	No. of times solution was given
($\underline{d}$; $\underline{r}$) where $\underline{d} \times \underline{r} = \underline{a} + \underline{b}$	1
($\underline{c}$; $\underline{b}$)	3
($\underline{a}$; $\underline{c}$)	2
($\underline{a}$; $\underline{a}$)	2
($\underline{b}$; $\underline{a}$)	2
($2\underline{a}$; $\underline{b} \times \underline{c}$)	1
($\underline{a} \times \underline{a}$; $\underline{b} \times \underline{c}$)	3
($\underline{b} \times \underline{c}$; $\underline{c} \times \underline{a}$)	2
($\underline{a}$; $\underline{b} - \underline{c}$)	1
($\underline{a}$; $\frac{1}{2}\underline{b} + \frac{1}{2}\underline{c}$)	1
($\underline{b} \times \underline{a}$; 1)	1
($\underline{b} \times \underline{a}$; $\overline{\underline{b}} \times \overline{\underline{a}} + \overline{\underline{c}} \times \overline{\underline{a}}$)	1
($\underline{a}$; $\underline{b} + \underline{c}$)	2
($\underline{b} - 10$; $\underline{a} - 10$)	1
($\underline{a}$; $\underline{b} + 1$)	1

[a] Solutions are represented by ordered pairs with $\underline{a}$, $\underline{b}$, and $\underline{c}$ representing whole numbers contained in distributive sentences presented to students ($\underline{a}$ represents the common factor while $\underline{b}$ and $\underline{c}$ represent the other two factors given in distributive sentences). Other small letters also represent whole numbers.

Computation Errors

Table 23 summarizes computation errors made by students when presented interview tests. An error in multiplication occurred 96 times, an error in addition occurred six times, and an error in factoring occurred eight times. All mutliplication errors were made by students attempting to find indicated products when using solution method 2 or solution method 3 (see Table 20 for a description of solution methods). When using solution method 2, students made a multiplication error four times, an addition error five times, and a factoring error eight times. These errors resulted in 13 incorrect solutions which accounted for 43.3% of the times solution method 2 was used. When using incorrect solution method 3, 92 errors were made in multiplication procedures. Since an incorrect solution would have resulted from use of solution method 3 anyway, only 13 incorrect solutions given during the study were attributable to computation errors.

Use of Incorrect Solution Methods

Students' incorrect solutions resulted primarily from use of incorrect solution methods (see Table 21). Use of incorrect solution methods accounted for 242 (91.3%) of the 265 incorrect solutions given during the investigation.[4] In particular, use of solution method 3 accounted for 198 (74.7%) of the 265 incorrect solutions. As a result of the frequent use of solution method 3, an effort was made to determine why students selected that solution method as often as they did. Some data resulting from this effort are presented in later sections of this chapter.

[4]Two of the 244 uses of incorrect solution methods resulted in correct solutions through "guessing."

TABLE 23

Summary of Computation Errors

Error category	Error code	No. of times error occured	No. of students making error at least once[b]	Error category	Error code	No. of times error occured	No. of students making error at least once[b]
Multiplication	1-a	8	3	Addition	2-a	1	1
	1-b	16	6		2-b	1	1
	1-d	7	3		2-e	3	2
	1-e	12	5		2-h	1	1
	1-g	12	10		Total	6	5
	1-h	13	4				
	1-i	6	5	Factoring	3	8	4
	1-k	22	15				
Total		96	51				

Note:--Error codes are explained in Directions for Using Interview Coding Form, Appendix A, Part III.
Some students made more than one kind of error.

Additional data resulting from this effort are presented in the following paragraphs.

Additional open sentences. In studying students' frequent use of solution method 3, the following two additional open sentences (a distributive and a non-distributive open sentence) were presented to each of 50 students following each student's interview test.

Open Sentence I: $(3 \times 4) + (3 \times 6) = (__ \times __)$

Open Sentence II: $(3 \times 4) + (3 \times 6) = (__ + __)$

The open sentences were presented on a single sheet of plain paper with one open sentence positioned immediately (double-spaced) below the other. The open sentences were randomly sequenced for presentation to each student with the common factor randomly positioned such that it was the first element in each indicated product for some students and it was the second element in each indicated product for other students. Thus, each of 50 students found a solution to each of the above open sentences.

Table 24 summarizes the frequency of various solutions given by the 50 students. For example, 29 students completed the open sentences as follows:

Open Sentence I: $(3 \times 4) + (3 \times 6) = (\underline{12} \times \underline{18})$

Open Sentence II: $(3 \times 4) + (3 \times 6) = (\underline{12} + \underline{18})$

Thirty-six of the 50 students placed the same numbers in the blanks as a solution for Open Sentence I as they did for Open Sentence II. Five students used different numbers but gave incorrect solutions to each of the two open sentences while nine students gave correct solutions to each of

TABLE 24

Summary of Student Solutions
for Additional Open Sentences

Description of solutions for Open Sentence I and II respectively	Number of students giving solution
12 x 18; 12 + 18	29
3 x 10; 3 + 10	3
Used same whole numbers a and b getting $a \times b$; $a + b$ (Both solutions incorrect)	4
Used different whole numbers a, b, c, and d getting $a \times b$; $c + d$ (Both solutions incorrect)	5
Used different whole numbers with both solutions correct (Used distributive principle when solving Open Sentence I	6
Used different whole numbers with both solutions correct (Did not use distributive principle)	3

Open Sentence I: $(3 \times 4) + (3 \times 6) = (__ \times __)$
Open Sentence II: $(3 \times 4) + (3 \times 6) = (__ + __)$

the two open sentences. Students correctly noted in each case that there was a difference between the two open sentences due to the change in operation sign on the right side of the equations.

Reading Difficulty

Students had considerable difficulty reading the four open distributive sentences correctly when attempting to read them the first time.[5] Forty-one of the 80 students made an error in reading at least one of the four open distributive sentences presented. When the 320 open distributive sentences were presented (four per student), 268 open distributive sentences were read correctly on the first attempt with 52 being read incorrectly. Table 25 summarizes errors made by students when attempting to read the open distributive sentences. For example, when reading open distributive sentences, an operation sign (either + or x) was skipped 10 times with a total of nine students making this particular reading error.

Relationship Between Performance and Factors

One purpose of the study was to investigate the relationship between elementary school students' performance when solving open distributive sentences of the form

$$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (__ \times __)$$

and

$$(\underline{b} \times \underline{a}) + (\underline{c} \times \underline{a}) = (__ \times __)$$

[5] Only data concerning students' reading of the open distributive sentences before attempting to solve them are reported due to learning effects.

TABLE 25

Errors Made by Students When Reading
Open Distributive Sentences

Error description	Number of times error occured	Number of students making error at least once
Reading of place-holder	20	20
Skipped a number	1	1
Skipped an operation sign	10	9
Skipped an equal sign	0	0
Misread a number	19	17
Misread an operation sign	12	12
Misread an equal sign	0	0
Other than above	0	0
Total	62	41[a]

[a]Some students made more than one reading error.

where a, b, and c represent whole numbers and three factors (Open Sentence Type, Context, Number Size) associated with these open sentences. Data and analysis of data used in achieving this purpose are presented in this section.

Each student's performance on each of four open distributive sentences presented on an individual interview test was represented by three response variates. These were 1) correct or incorrect solution, 2) use or non-use of the distributive principle, and 3) use or non-use of incorrect solution method 3. Students' frequent use of incorrect solution method 3 (see Table 21) indicated a need to check the effect of factors (Open Sentence Type, Context, Number Size) on the frequent selection of this incorrect solution method. The response variate CS (correct solutions) was assigned a 1 if a correct solution was given and a 0 if an incorrect solution was given, the response variate DP (use of the distributive principle) was assigned a 1 if the distributive principle was used as a solution method and a 0 otherwise, and the response variate IM (use of incorrect solution method 3) was assigned a 1 if solution method 3 was used and a 0 otherwise. For example, if a student gave an incorrect solution through incorrect use of the distributive principle when presented an open distributive sentence, the response vector [CS, DP, IM] would be [0, 1, 0]. Data supported the lack of differential effects due to sex when t tests were used with the independent variable being mean number of correct solutions and with the independent variable being mean number of times the distributive principle was used. As a result, sex was not used as a blocking variable in the following analyses.

Also, recognition level was not used as a blocking variable (see discussion in section Relationship Between Performance and Recognition Level Groups later in this chapter). Although the relatively low student performance level and the relatively low performance variance strongly suggested that no differential effects would be found in connection with the principal variables investigated, the data were analyzed as planned to confirm this suspicion.

Open Sentence Type

Each of the 80 students were presented two open distributive sentences reflecting the left-horizontal (L-H) open sentence type and two open distributive sentences reflecting the right-horizontal (R-H) open sentence type. Table 26 summarizes performance data for the two open sentence types. For example, 80 students provided 27 correct solutions (a mean of 0.34 per student) when presented 160 L-H open distributive sentences (two per student). The percent of total correct solutions (% of 55) given to L-H open distributive sentences by 17 students was 49.1. The mean response vector for student performance on L-H open distributive sentences was [0.34, 0.28, 1.26] and the mean response vector for student performance on R-H open distributive sentences was [0.35, 0.30, 1.21]. The results of the ANOVAs (analysis of variance tests) used to test the difference between the mean response variates for open sentence types were not significant ($\alpha = .05$) (see Table 27). The data support the null hypotheses of no differential performance due to open sentence type when each response variate is considered as the dependent variable.

TABLE 26

Summary of Responses
by
Open Sentence Type

Open sentence type	Response variate	Number of students	Number of students giving at least one response coded 1	Number of responses coded 1	% of total responses coded 1	Mean number of responses coded 1 per student
L-H	CS	80	17	27	49.1	0.34
	DP	80	12	22	47.9	0.28
	IM	80	53	101	51.0	1.26
R-H	CS	80	18	28	50.9	0.35
	DP	80	13	24	52.1	0.30
	IM	80	52	97	49.0	1.21

L-H: Left-Horizontal
R-H: Right-Horizontal

CS: Correct Solutions
DP: Use of Distributive Principle
IM: Use of Solution Method 3

TABLE 27

Summary of ANOVAs
for an
Open Sentence Type Main Effect

Hypothesis	Variable	Error	MS	df	F-ratio	p <
u(L-H) = u(R-H)	CS	0.016	0.003	(1,76)	0.196	0.66
	DP	0.006	0.013	(1,76)	2.000	0.161
	IM	0.025	0.050	(1,76)	2.027	0.159

Context

Twenty of the 80 students were presented interview tests in the symbolic (S) context, 20 were presented interview tests in the symbolic and pictorial (SP) context, 20 were presented interview tests in the symbolic and verbal (SV) context, and 20 were presented interview tests in the symbolic, pictorial, and verbal (SPV) context. Table 28 summarizes performance data for the four contexts. For example, 20 students who were presented 80 open distributive sentences (four per student) in the symbolic context provided 18 correct solutions (a mean of 0.90 per student). The percent of total correct solutions (% of 55) given by six students to open distributive sentences presented in the symbolic context was 32.7. The mean response vectors for student performance when open distributive sentences were presented in the S, SP, SV, and SPV contexts were [0.90, 0.80, 1.75], [0.85, 0.75, 2.35], (0.95, 0.75, 2.70], and [0.05, 0.00, 3.10], respectively. Results of the ANOVAs used to test the difference between the mean response variates for contexts were not significant ($\alpha = 0.05$) (see Table 29). Data support the null hypotheses of no differential performance due to context when each response variate is considered as the dependent variable.

Number Size

Each of the 80 students was presented two open distributive sentences reflecting large number size (L) and two open distributive sentences reflecting small number size (T). Table 30 summarizes data for the two number sizes. For example, the 80 students who were presented 160 open distributive sentences (two per student) containing large numbers provided

TABLE 28

Summary of Responses
by
Context

Context	Response variate	No. of students	No. of students giving at least one response coded 1	No. of responses coded 1	% of total responses coded 1	Mean no. of responses coded 1 per student
S	CS	20	6	18	32.7	0.90
	DP	20	4	16	34.8	0.80
	IM	20	11	35	17.7	1.75
SP	CS	20	7	17	30.9	0.85
	DP	20	4	15	32.6	0.75
	IM	20	13	47	23.7	2.35
SV	CS	20	6	19	34.6	0.95
	DP	20	5	15	32.6	0.75
	IM	20	14	54	27.3	2.70
SPV	CS	20	1	1	1.8	0.05
	DP	20	0	0	0.0	0.00
	IM	20	16	62	31.3	3.10

S: symbolic
SP: symbolic & pictorial
SV: symbolic & verbal
SPV: symbolic, pictorial, & verbal

CS: correct solutions
DP: use of distributive principle
IM: use of solution method 3

TABLE 29

Summary of ANOVAs
for a
Context Main Effect

Hypothesis	Variable	Error	MS	df	F-ratio	p<
u(S) = u(SP) = u(SV) = u(SPV)	CS	0.448	0.912	(3,76)	2.034	0.116
	DP	0.456	0.738	(3,76)	1.616	0.193
	IM	0.823	1.638	(3,76)	1.989	0.123

TABLE 30

Summary of Responses
by
Number Size

Number size	Response variate	Number of Students	Number of students giving at least one response coded 1	Number of responses coded 1	% of total responses coded 1	Mean number of responses coded 1 per student
L	CS	80	15	26	47.3	0.33
	DP	80	12	23	50.0	0.29
	IM	80	49	94	47.5	1.18
T	CS	80	17	29	52.7	0.36
	DP	80	12	23	50.0	0.29
	IM	80	53	104	52.5	1.30

L: Large
T: Small

CS: Correct Solutions
DP: Use of Distributive Principle
IM: Use of Solution Method 3

26 correct solutions (a mean of 0.33 per student). The percent of total correct solutions (% of 55) given by 15 students to open distributive sentences containing large numbers was 47.3. The mean response vector for student performance on open distributive sentences containing large numbers was [0.33, 0.29, 1.18] and the mean response vector for student performance on open distributive sentences containing small numbers was [0.36, 0.29, 1.30]. Univariate tests (ANOVAs) indicate that performance data support the null hypothesis of no difference in mean number of correct solutions due to number size and the null hypothesis of no difference in mean number of times the distributive principle was used due to number size. However, data do not support the null hypothesis of no difference in mean number of times solution method 3 was used due to number size ($\alpha = 0.05$). Data indicate that students selected solution method 3 more often when open distributive sentences containing small numbers were presented than when open distributive sentences containing large numbers were presented. Results of the univariate tests are summarized in Table 31.

Interactions of Factors

A summary of the ANOVAs used in testing for existence of interactions between the three factors (Open Sentence Type, Context, Number Size) reflected by data is presented in Table 32. Results indicate that data do not support existence of interactions between the three factors.

Relationship Between Performance and Recognition Level Groups

One purpose of the study was to investigate the relationship between elementary school students' use of the distributive principle as a method

TABLE 31

Summary of ANOVAs
for a
Number Size Main Effect

Hypothesis	Variable	Error	MS	df	F-ratio	p<
u(L) = u(T)	CS	0.055	0.028	(1,76)	0.514	0.476
	DP	0.019	0.000	(1,76)	0.000	1.000
	IM	0.060	0.313	(1,76)	5.191	0.036*

TABLE 32

Summary of ANOVAs
for
Interactions

Hypothesis	Variable	Error	MS	df	F-ratio	p<
C X NS	CS	0.055	0.020	(3,76)	0.361	0.781
	DP	0.019	0.008	(3,76)	0.429	0.733
	IM	0.060	0.038	(3,76)	0.623	0.602
C X OST	CS	0.016	0.012	(3,76)	0.718	0.544
	DP	0.006	0.004	(3,76)	0.667	0.575
	IM	0.025	0.025	(3,76)	1.013	0.392
NS X OST	CS	0.040	0.078	(1,76)	1.971	0.165
	DP	0.006	0.000	(1,76)	0.000	1.000
	IM	0.013	0.000	(1,76)	0.000	1.000
C X NS X OST	CS	0.040	0.053	(3,76)	1.340	0.268
	DP	0.006	0.008	(3,76)	1.333	0.270
	IM	0.013	0.008	(3,76)	0.650	0.586

C: Context; NS: Number Size; OST: Open Sentence Type

for solving the open distributive sentences and selected levels of students' recognition of the distributive principle. The low reliability of the Recognition Level Test (see Reliability of Recognition Level Test, Chapter III) implies that one cannot discriminate confidently between the four selected recognition levels. Due to low reliability of the Recognition Level Test and due to students' lack of using the distributive principle to any great extent, the relationship between students' use of the distributive principle and students' level of recognizing the distributive principle could not be determined with any reasonable degree of confidence. The intended analyses pertaining to performance level and recognition level group were not carried out.

Characteristics of Students and Contexts

Due to students' lack of using the distributive principle when open distributive sentences were presented in the SPV context, the following data regarding characteristics of students presented interview tests in various contexts are presented. The mean IQs of students presented open distributive sentences in the S, SP, SV, and SPV contexts were 108.0, 106.8, 112.2, and 110.0, respectively, the mean mathematics achievement scores were 27.4, 28.85, 26.5, and 26.0, respectively, and the mean concept attainment scores were 9.5, 10.8, 10.2, and 10.2, respectively (see Characteristics of Students in Chapter IV for information regarding tests used in obtaining IQs, mathematics achievement scores, and concept attainment scores). Table 33 presents the number of students from each school who were presented open distributive sentences in various contexts.

TABLE 33

Number of Students per School Within Each Context

Context	School			Total
	1	2	3	
S	10	3	7	20
SP	6	7	7	20
SV	4	5	11	20
SPV	9	2	9	20
Total	29	17	34	80

Characteristics of Students Using the Distributive Principle

Table 34 presents characteristics of the 13 students who attempted to use the distributive principle at least once when presented four open distributive sentences. A total of eight students (five girls and three boys) used the distributive principle correctly four times each. The mean IQ of the eight students was 114, the mean mathematics achievement was 34.5 (50 possible), and the mean concept attainment was 11.125 (17 possible). Nine of the 13 students who attempted to use the distributive principle at least once was from school 3. Six of the eight students using the distributive principle when solving each of the four open distributive sentences presented on an interview test were from school 3. A chi-square test of association between students' use of the distributive principle and school attended resulted in a p-value less than 0.09 with a sample phi coefficient estimating the strength of association as 0.24 (see Table 35).

Summary

Chapter V has presented data collected and used in achieving the fourfold purpose of the investigation and results of tests used in analyzing the data. Chapter VI presents a summary of the investigation, a discussion of delimitations of the investigation, conclusions resulting from the investigation, and implications of the investigation.

TABLE 34

Characteristics of Students Using the Distributive Principle

Student no.	Sex	IQ	MA	CA	School	No. of times used the dis. principle	No. of correct solutions
09	F	127	40	14	1	4	0
19	F	100	25	7	3	4	4
11	F	118	43	13	2	4	4
16	M	114	38	16	3	4	4
14	F	111	34	12	3	2	2
15	F	95	14	6	3	1	1
17	M	124	39	11	3	4	4
34	M	124	36	12	3	4	4
30	F	126	37	9	2	4	4
52	F	104	34	13	3	4	4
54	F	100	24	8	3	4	4
76	F	109	24	8	3	4	0
61	M	112	37	8	1	3	1

MA: mathematics achievement; CA: concept attainment

TABLE 35

Summary of Chi-Square Test for Association Between Use of Distributive Principle and Schools Attended

School	No. of students observed using D. Prin. at least once	No. of students observed not using D. Prin. at least once	Total
1	2	27	29
2	2	15	17
3	9	25	34
Total	13	67	80

$X^2 = 4.73$; df = 2; $p < 0.09$; $\hat{\phi} = 0.24$

Chapter VI

CONCLUSIONS AND IMPLICATIONS

The fourfold purpose of this investigation was

1. to investigate the relationship between elementary school students' performance when solving open distributive sentences of the form

$$(\underline{a} \times \underline{b}) + (\underline{a} \times \underline{c}) = (___ \times ___)$$

and

$$(\underline{b} \times \underline{a}) + (\underline{c} \times \underline{a}) = (__ \times __)$$

where $\underline{a}$, $\underline{b}$, and $\underline{c}$ represent whole numbers and selected levels of three factors (Open Sentence Type, Context, Number Size) associated with these open sentences,

2. to investigate the relationship between elementary school students' use of the distributive principle as a method for solving the open distributive sentences and selected levels of students' recognition of the distributive principle,

3. to identify and classify other methods attempted by elementary school students when solving the open distributive sentences, and

4. to identify and classify errors in elementary school students' performance when solving the open distributive sentences.

Student performance on a recognition level test was used to assign each of 301 fifth-grade students to a level of recognition of the distributive principle. Four recognition level groups with each group of students representing a different level of recognition were selected to participate in the investigation. Twenty students from each of the four groups were selected as subjects.

Each student was presented an interview test consisting of four open distributive sentences of the form indicated above with each open sentence reflecting a different combination of the four possible combinations of two levels of open sentence type [left-horizontal (L-H); right-horizontal (R-H)] and two levels of number size [large (L); small (T)]. Each student's interview test was presented in one of four context levels [symbolic (S); symbolic and pictorial (SP); symbolic and verbal (SV); symbolic, pictorial, and verbal (SPV)]. The performance of each student on a four-item interview test and responses given during an individual interview provided data for use in achieving the fourfold purpose of the investigation. The data collected and used are reported and analyzed in Chapter V. The following sections of this chapter present delimitations, conclusions, and implications of the investigation.

Delimitations of Conclusions and Implications

Conclusions and implications of the investigation cannot be generalized with assurance beyond the population of students from which the sample of 80 students were selected. Generalizations of conclusions and implications of the present investigation to a given population of students are valid to the extent that characteristics of that population coincide with characteristics of the population from which the sample of students participating in the investigation was drawn and to the extent that the conclusions and implications are applied to performance of the given population in solving open distributive sentences of the form investigated.

Conclusions and Comments

Statistical Conclusions

Effect of Factors on Student Performance

Since no significant differences were observed between boys' and girls' overall performance, these findings are based on data pooled across sexes.

1. Data support the null hypothesis of no differential performance between the mean number of correct solutions given for left-horizontal open distributive sentences and the mean number of correct solutions given for right-horizontal open distributive sentences.

2. Data support the null hypothesis of no differential performance between the mean number of times the distributive principle was used when left-horizontal open distributive sentences were presented and the mean number of times the distributive principle was used when right-horizontal open distributive sentences were presented.

3. Data support the null hypothesis of no differential performance between the mean number of times solution method 3 was used when left-horizontal open distributive sentences were presented and the mean number of times solution method 3 was used when right-horizontal open distributive sentences were presented.

4. Data support the null hypothesis of no differential performance in mean number of correct solutions due to context when open distributive sentences were presented in the four contexts (S, SP, SV, SPV).

5. Data support the null hypothesis of no differential performance in mean number of times the distributive principle was used due to context when open distributive sentences were presented in the four contexts (S, SP, SV, SPV).

6. Data support the null hypothesis of no differential performance in mean number of times solution method 3 was used due to context when open distributive sentences were presented in the four contexts (S, SP, SV, SPV).

7. Data support the null hypothesis of no differential performance between the mean number of correct solutions given for open distributive sentences containing large numbers and the mean number of correct solutions given for open distributive sentences containing small numbers.

8. Data support the null hypothesis of no differential performance between the mean number of times the distributive principle was used when open distributive sentences containing large numbers were presented and the mean number of times the distributive principle was used when open distributive sentences containing small numbers were presented.

9. Data did not support the null hypothesis of no differential performance between the mean number of times incorrect solution method 3 was used when open distributive sentences containing large numbers were presented and the mean number of times solution method 3 was used when open distributive sentences containing small numbers were presented.

10. Data did not support existence of an interaction between Open Sentence Type and Context when either mean number of correct solutions, mean number of times the distributive principle was used, or mean number of times solution method 3 was used was considered as the dependent variable.

11. Data did not support existence of an interaction between Open Sentence Type and Number Size when either mean number of correct solutions, mean number of times the distributive principle was used, or mean number of times solution method 3 was used was considered as the dependent variable.

12. Data did not support existence of an interaction between Context and Number Size when either mean number of correct solutions, mean number of times the distributive principle was used, or mean number of times solution method 3 was used was considered as the dependent variable.

13. Data did not support existence of an interaction between Open Sentence Type, Context, and Number Size when either mean number of correct solutions, mean number of times the distributive principle was used, or mean number of times solution method 3 was used was considered as the dependent variable.

Effect of Schools[1] Attended on Use of Distributivity

14. Data did support existence of an association between school attended and student performance when the dependent variable was number of students using the distributive principle at least one time when attempting to find solutions to interview test items ($\alpha = 0.10$)!

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[1] See Comments on Effect of Schools Attended on Use of Distributivity in this chapter.

Comments

Statistical conclusions 1-13 provide information about the relationship between elementary school students' performance when solving open distributive sentences of the form presented during the investigation and selected levels of three factors (Open Sentence Type, Context, Number Size) associated with the open sentences; statistical conclusion 14 provides additional information discussed in a following section.

Comments of Effect of Factors on Student Performance

Open Sentence Type. Statistical conclusions do not indicate significant differential student performance due to open sentence type when the dependent variables are number of correct solutions, use of the distributive principle, and use of incorrect solution method 3. The author finds no reason to suspect that individual students react differently in performance when presented left-horizontal or right-horizontal open distributive sentences. Evidence provided by this investigation is consistent with evidence presented by Weaver (1973c). Weaver's data[2] did not indicate existence of differential effects on student performance in solving open distributive sentences due to the positioning of the common factor (i.e., left or right distributivity) in the multiplication-addition context (the multiplication-addition context was the same as the symbolic context used in the present study except Weaver's investigation included a vertical as well as a horizontal format). It is noted that Weaver's evidence applies to open distributive sentences of different form (format) than those studied in the present investigation (see Table 2, Chapter II).

[2]Weaver's data were not subjected to extensive statistical analysis.

It is also noted that Weaver's data did indicate existence of possible differential effects on student performance in a set analog context due to positioning of the common factor.

Context. Statistical conclusions do not indicate significant differential student performance due to context when the dependent variables are number of correct solutions, use of the distributive principle, and use of solution method 3. The relative poor performance in number of correct solutions, number of times the distributive principle was used, and number of times solution method 3 was used when open distributive sentences were presented in the SPV context (see Table 28 in Chapter V) may lead one to suspect the statistical conclusion of no differential effects due to context. For example, when open distributive sentences were presented in the S, SP, SV, and SPV contexts, the number of times students used the distributive principle was 16, 15, 15, and 0 times, respectively. However, the small number of students using the distributive principle within each context resulted in a mean of less than one time per student that the distributive principle was used in each context. Also, students' consistent lack of using the pictorial or verbal cuing when open distributive sentences were presented in the SP, SV, and SPV contexts during the investigation substantiates the statistical conclusion. The author has no reason to suspect the statistical conclusion based on collected data.

When seeking to find a plausible reason for the relative poor performance by students presented open distributive sentences in the SPV context, the characteristics of students presented open distributive

sentences in each context were examined.[3] Differential performance due to sex and recognition levels were balanced between context levels and differential performance due to IQ, mathematics achievement, concept attainment, and schools attended does not seem likely. However, some variable(s) affecting student performance when open distributive sentences were presented in the SPV context may have escaped investigation (e.g., the increase in verbal directions during interviews when open distributive sentences were presented in the SPV context as opposed to verbal directions when open distributive sentences were presented in the remaining three contexts).

Number Size. Statistical conclusions do not indicate differential performance due to number size when the dependent variables are number of correct solutions and use of the distributive principle. However, data do support differential performance due to number size when the dependent variable is use of solution method 3. Data indicate that slightly more students used solution method 3 (see Table 20 for description of solution method 3) when open distributive sentences containing small numbers were presented. However, this situation resulted mainly because some students used solution method 3 when solving open distributive sentences containing small numbers and used solution method 6 procedures (see Table 20) when solving open distributive sentences containing large numbers. The change in solution procedures was due primarily to a lack of students' ability to multiply large numbers (two-digit numbers).

Interactions. Collection of data concerning the conceptual problem referred to by Weaver (see discussion of context levels in Chapter II)

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[3]See Characteristics of Students and Contexts in Chapter V.

when students were presented equations of the form

_a_ sets of _b_ and _a_ sets of _c_ are ___ sets of ___

and

_b_ sets of _a_ and _c_ sets of _a_ are ___ sets of ___

where _a_, _b_, and _c_ are whole numbers was limited due to students' failure to use the distributive principle and sets to any great degree. The conceptual problem referred to would seem to imply that students who used sets to find a solution for open distributive sentences presented in the SP or SPV context would have more difficulty when presented left-horizontal open distributive sentences than when presented right-horizontal open distributive sentences (i.e., if sets were used in the analogous manner usually thought indicative of the distributive idea). Students ignored the sets when using or not using the distributive principle except for a few isolated cases when students used sets to check multiplication procedures. At no time did students use sets in a manner indicative of recognition that they could be joined as desired when representing the distributive idea. Performance by students provides no evidence contrary to the statistical conclusion that data do not support existence of an Open Sentence Type by Context interaction. Likewise, the author finds no reason to suspect that statistical conclusions regarding other interactions investigated.

Comments on Effect of Schools Attended on Use of Distributivity

Each student who participated in the investigation was selected from one of three elementary schools. It should be noted that data did

support a statistical association between school attended and number of students using the distributive principle for $\alpha = 0.10$ (see Table 35, Chapter V). Raw data do appear to support this association between school attended and number of students using the distributive principle. Nine of the 13 students who attempted to use the distributive principle at least one time during the investigation were students who attended school 3. Differences and similarities in mathematics programs in the three schools need considering and are discussed in Chapter IV (see Characteristics of Student Sample section on Opportunity to Learn).

Comments on Identification and Classification of Solution Methods

Solution methods used by students during the investigation were identified and classified according to the scheme summarized in Table 20 (Chapter V). A summary of the number of students using each solution method at least one time, the total number of times each solution method was used by students during the investigation, the percent of total times possible that each solution method was used, and the mean number of times per student that each solution method was used is presented in Table 21 (Chapter V). Since solution method 6 and solution method 8 were used to designate a variety of solution procedures, students' solution procedures which were identified as one of these two solution methods are discussed in Chapter V. Table 22 (Chapter V) presents a summary of solutions resulting from procedures identified as solution method 8. The frequent use of incorrect solution methods during the investigation led to many incorrect solutions. Therefore, students' use of incorrect solution methods is discussed in the following section.

Comments on Identification and Classification of Errors

Identification, classification, and a discussion of errors reflected by students' performance are presented in Chapter V (see section on Errors). The use of incorrect solution methods accounted for 242 (91.3%) of the 265 incorrect solutions given by students during the investigation.[4] Ten of the remaining 23 incorrect solutions resulted from students' incorrect use of the distributive principle and 13 incorrect solutions resulted from computational errors in multiplication, addition, or factoring when students used solution method 2 (see Table 20 for a description of solution methods and Table 23 for a listing of computational errors).

Incorrect solution method 3 was selected by students 198 times with only 16 students not selecting method 3 at least one time during performance on interview tests. The frequent selection of this incorrect solution method accounted for 74.7% of the 265 incorrect solutions. The frequent occurrence of this error (selection of solution method 3) was of concern to the author and a limited exploration of students' reason(s) for selecting this solution method was undertaken.

Difficulty in reading the open distributive sentences seemed to be a plausible reason for selection of solution method 3 by students (e.g., students could read the operation sign on the right side of the open distributive sentences as an addition sign rather than a multiplication sign). Data on difficulty students had in reading the open distributive sentences were collected during the investigation and are summarized in Table 25 (Chapter V). The data do reflect that students have difficulty

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[4] Actually, incorrect solution methods were used a total of 244 times with 2 correct solutions resulting from procedures classified as solution method 8 and characterized as "guessing."

in reading the open distributive sentences. This difficulty could certainly affect student performance when students are individually solving open distributive sentences. However, since the author had students read each open sentence correctly before attempting to find a solution, reading difficulty should not have been a major reason for students' frequent selection of solution method 3. Performance by 50 students who were presented two additional open sentences (see Chapter V, Additional open sentences) provided more evidence that reading difficulty is not the major reason for students' frequent selection of solution method 3.

Results of statistical tests indicate that data do not support significant difference in the mean number of times students select solution method 3 due to Open Sentence Type, Context, and/or interactions between the three factors (Open Sentence Type, Context, Number Size). Data do indicate a significant difference ($\alpha = 0.10$) in the mean number of times students select solution method 3 due to number size (see Comments on Effect of Factors on Student Performance in a previous section of this chapter).

Implications

Implications of Effect of Factors on Student Performance

No significant differences in student performance due to Open Sentence Type, Context, and/or interactions between the three factors (Open Sentence Type, Context, Number Size) were found when correct solutions, use of the distributive principle, and use of incorrect solution method 3 were considered as dependent variables. No significant difference in

mean number of correct solutions and in mean number of times the distributive principle was used as a solution method was found due to Number Size. Data did support a significant difference in mean number of times solution method 3 was used due to number size. However, one should be cautious when interpreting and generalizing these results (see Delimitations of Conclusions and Implications in a previous section of this chapter). Three topics do seem to deserve special attention: 1) students' lack of using available information when open distributive sentences were presented in the SP, SV, and SPV contexts, 2) a conceptual problem to which collected data should not be applied, and 3) the relationship between student performance and number size.

First, students' lack of using available information when open distributive sentences were presented can not be attributed to students' not needing to use the information. If the verbal cue (that students could use the distributive principle) which was given when open distributive sentences were presented in the SV and SPV contexts had been used correctly by students, performance would certainly have improved. A similar statement could be made concerning the pictorial cue given when open distributive sentences were presented in the SP and SPV contexts. Students' lack of using available information was probably a result of many not knowing how to use the information. Students participating in the investigation had been exposed to the distributive idea using terminology and sets as presented in the investigation each year since and including the third-grade level (see Table 17, Chapter IV).[5] If objectives of the elementary

[5]The Houghton-Mifflin series (Duncan et al., 1970) was used during students' fourth- and fifth-grade levels. The Greater Cleveland Mathematics Program (SRA, 1968) was used during students' third-grade level.

school mathematics program[6] are simply to introduce the analogy between joining of sets and the distributive idea and simply to introduce terminology related to the distributive idea, these objectives have been achieved. On the other hand, if objectives of the mathematics program are to prepare fifth-grade students so they recognize the analogy between joining of sets and the distributive idea and so they use this analogy when solving open distributive sentences of the form presented in this investigation and/or if objectives are to prepare fifth-grade students so they follow a verbal cue similar to that presented in the investigation when solving open distributive sentences of the form presented in the investigation, then these objectives have not been achieved. The above statements apply to the mathematics program used by students participating in the investigation and similar programs.

Second, data collected in this investigation do not either support or not support existence of the possible conceptual problem referred to in earlier discussions (see previous comments on Interactions in this chapter). For example, the conceptual problem (if it exists) would probably cause differential student performance only if students attempted to use the sets when solving the open distributive sentences. Since students participating in the investigation did not use the sets, collected data should not be used in determining support or non-support of differential student performance resulting from the possible conceptual problem. Further research is needed in this particular area.

Third, student performance when finding correct solutions and when applying the distributive principle to find solutions to the open

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[6]Program is used in a broad sense to include instructional materials and instructional procedures.

distributive sentences was not affected by number size. However, there does seem to be a small tendency for more students to use incorrect solution method 3 when open distributive sentences containing small numbers are presented than when open distributive sentences containing large numbers are presented. This tendency probably occurs due to the ease with which students recall many basic multiplication facts and the difficulty students have in multiplying two-digit numbers. It very well may be that introduction to the distributive principle using open distributive sentences containing large numbers (numbers greater than 10) prior to students learning to multiply two-digit numbers would be beneficial for at least two reasons:

1. students would have difficulty in applying incorrect solution method 3, and
2. students could realize a saving of labor in using the distributive principle.

Introduction of the distributive principle using open distributive sentences containing large numbers as opposed to small numbers (numbers between 0 and 10) needs investigation.

Implications of Effect of Schools Attended on Student Performance

Nine of the 13 students who used the distributive principle at least one time during the investigation were from the same school (school 3). Information concerning mathematics programs at the three elementary schools (see discussion on Schools and Opportunity to Learn in Chapter IV) indicate the following differences which could account for more students from school 3 using the distributive principle than from the other two schools.

1. School 3 had instituted an individualized mathematics program including use of 30 individualized student packets. This program may have increased students' need to ask questions and to seek meaningful answers (i.e., the student who recognized the distributive principle at levels from which students were selected to participate in the investigation may have been the initiator of the "questioning and information getting" process while students attending the mathematics programs at the other two schools [traditional classroom situations as opposed to the individualized program at school 3] may not have been "initiators" of the process to the extent that students attending school 3 were). Research is certainly needed in studying the variable mentioned above (individualized vs. traditional involvement of students in the "questioning and information getting" process). Other variables may also need consideration.

2. School 3 had just initiated the individualized program (i.e., this was the first year students had participated in the individualized program). A "Hawthorne Effect" may have been in operation at the school due to the new procedures.

3. The individualized mathematics program at school 3 contained a more extensive review of third- and fourth-grade material before students were presented fifth-grade material as opposed to the mathematics programs at the other two schools (for a brief description, see Opportunity to Learn in Chapter IV).

Certainly the difference in performance by students presented different mathematics programs merits further study.

Implications of Students' Use of Solution Methods

Students in general did not try to determine if a solution given to an open distributive sentence during the investigation was a correct or an incorrect solution. Therefore, students in general did not individually determine a correct solution method from an incorrect solution method. The lack of students' use of checking procedures may indicate an inability which could be a major factor in students' frequent use of incorrect solution methods when presented items on the interview tests. Students may not know how to distinguish between correct and incorrect solution methods

due to an inability to check solutions. If an objective of the elementary school mathematics program is to prepare fifth-grade students so they can individually find solution methods for open distributive sentences of the form presented on interview tests, that objective has been achieved. If an objective of the mathematics program is to prepare fifth-grade students so they can individually find solution methods for the open distributive sentences and determine if the solution methods are correct or incorrect (i.e., if the solution methods provide correct or incorrect solutions), then that objective does not seem to have been achieved. If the latter objective is the one desired, then the mathematics program needs reconsidering. As mentioned earlier, these statements apply to programs similar to those presented to students who participated in the investigation.

Implications of Students' Errors

Although students did make computational errors (especially when multiplying two-digit numbers), the major error was the frequent selection of incorrect solution methods and in particular the frequent selection of solution method 3. The frequent use of solution method 3 certainly merits further investigation. Students do have difficulty in correctly reading open distributive sentences of the form presented during the investigation and this difficulty also needs attention. However, this difficulty in reading open distributive sentences is not the major reason students select incorrect solution method 3. A major reason may be that students simply don't know how to distinguish between a correct and an incorrect solution method. Other alternative explanations exist and need investigation.

Due to the frequent selection of incorrect solution methods, computation errors did not account for many incorrect solutions. Solution method 2 was selected for use a total of 30 times and computational errors were made 13 of those times (i.e., 43.3% of the times correct solution method 2 was used, computation errors prevented students from obtaining correct solutions). The abundance of computation errors certainly indicates a need to reconsider the elementary school mathematics program if an objective of the program is to prepare fifth-grade students so they can multiply, add, and factor with the competency required to adequately use solution method 2.

Implications for Future Research

Previous sections have presented (directly and indirectly) a number of implications for future research. A few of these implications are summarized in this section along with other implications for future research. Delimitations of implications should be observed.

1. If an objective of elementary school mathematics programs is to present to students an introduction of the distributive principle which results in fifth-grade students who apply the distributive principle when solving open distributive sentences of the form presented during this investigation, current instructional materials and/or procedures are not attaining this objective. New instructional materials and/or procedures may need developing or if needed materials and procedures are already developed, they need to be used correctly. What these materials and/or procedures are and how to use them correctly need to be investigated.

2. If an objective of elementary school mathematics programs is to facilitate fifth-grade students' understanding of the distributive principle through an analogous procedure used in joining sets or arrays, then new instructional materials and/or procedures may need developing or if needed materials and procedures are already developed, they need to be used correctly. What these materials and/or procedures are and how to use them correctly need to be investigated.

3. If an objective of elementary school mathematics programs is fifth-grade students who can "meaningfully" attach to the distributive principle the name given to it, then new instructional materials and/or procedures may need developing or if needed materials and/or procedures are already developed, they need to be used correctly. What these materials and/or procedures are and how to use them correctly need to be investigated.

4. If an objective of elementary school mathematics programs is fifth-grade students who accurately evaluate solution methods used when solving open distributive sentences of the form presented in this investigation through checking procedures, then new instructional materials and/or procedures may need developing or if needed materials and/or procedures are already developed, they need to be used correctly. What these materials and/or procedures are and how to use them correctly need to be investigated.

5. A number of questions such as the following could be stated and these questions need further investigation. Should the distributive principle be introduced in the elementary school mathematics curriculum? If not, when should it be introduced? If it should be, when? How? To what extent? Which kind of open distributive sentences should be used? . . . ?

6. The generalizability of results from the present investigation is limited. Do students who are presented a different mathematics program from the programs presented students in this investigation recognize and use the distributive principle with similar performance as students who participated in this investigation? This investigation provides evidence that they may not (see discussion on Implications of Effect of Schools Attended on Student Performance in this chapter). Would presentation of open distributive sentences unlike those presented in the present investigation result in different student performance? Other questions concerning generalizability of results could be generated and need investigating.

7. If elementary school students are having difficulty of the magnitude indicated by this investigation, by the author's pilot study (1973), and by Weaver's large scale investigation (1973c), in recognizing and using the distributive principle, then what about elementary school students' difficulty in recognition and use of other ideas such as identity elements, inverse elements, commutativity, and associativity? These areas certainly need investigating.

8. Student performance on open sentences (not necessarily open distributive sentences) and the effect of certain factors on that performance is a fertile ground for exploration. Prior to this investigation, only four recent investigations [Weaver (1971, 1973c); Grouws (1971); Hobbs (1973)] have made an effort to determine the effect of certain factors associated with open sentences on student performance. Since open sentences are instrumental in introducing most ideas in current elementary school mathematics programs, further investigation is needed in this area.

9. The introduction of the distributive principle to elementary school students as early as third-grade level seems to have occurred with limited research (if any) to support this early introduction; the use of an analogous joining of sets or joining of arrays procedure seems to have been introduced with the idea that students could transfer the analogous procedure used in joining the sets or arrays to the symbolic procedure using distributivity when solving open distributive sentences with limited research (if any) to support this idea; the use of a variety of open distributive sentences are presented in current elementary school mathematics programs with little apparent consideration given to the limited amount of research related to student performance in solving open sentences.[7] Similar statements could be made concerning the limited amount of research related to a variety of mathematical ideas presented in current elementary school mathematics programs. The above statements imply that one of the major implications for future research and for persons responsible for elementary school mathematics programs is that changes in instructional materials and procedures should be based on previous research rather than research being based on previous changes in instructional materials and procedures.

In conclusion, elementary school students' lack of using the distributive principle in the present investigation echoes the results of Weaver (1973c) and Hobbs (1973). Results do not indicate if elementary school students do or do not have an informal "idea" about distributivity and cannot recognize and/or use this principle in the formal ways presented

[7]These statements are true for elementary school mathematics programs in general but not necessarily true for all elementary school mathematics programs [e.g., Developing Mathematical Processes (Wisconsin Research and Development Center for Cognitive Learning) is an elementary mathematics program based on research].

in the investigation. A major implication of these investigations is that the pedagogical reason for teaching the distributive principle may not hold true for elementary school students. The postponement of a formal introduction to the distributive principle until students reach a higher level of mathematical maturity may very well be a wise decision to make. If this postponement occurred, implications for a change in rationalization and introduction of multiplication algorithms in the elementary school must be considered.

APPENDIX A

INVESTIGATION INSTRUMENTS

Part I

Recognition Level Test

WHICH

LETTER?

THIS BOOKLET CONTAINS 28 QUESTIONS TO BE ANSWERED. SOME POSSIBLE ANSWERS ARE GIVEN BELOW EACH QUESTION. ONLY ONE OF THE ANSWERS GIVEN IS A CORRECT ANSWER. READ EACH QUESTION CAREFULLY. DECIDE WHICH OF THE GIVEN ANSWERS IS THE CORRECT ANSWER. FIND THE LETTER THAT HAS BEEN ASSIGNED TO THAT ANSWER AND MARK THAT LETTER ON THE ANSWER SHEET. WE'LL WORK THE FIRST 3 QUESTIONS TOGETHER.

1. Which of the following is <u>always</u> true for a place holder?

   A. It holds a place for a number.

   B. It is an X.

   C. It is a box.

2. Which of the following does <u>NOT</u> show subtraction?

   A. 5 children, 2 of which are girls. The number of boys is 3.

   B. $4 - 2 = 2$.

   C. $4 + 1 = 5$.

3. Which of the following is another name for the common factor property?

   A. identity property

   B. closure property

   C. distributive property

4. The set { } is an example of:

   A. an open set.

   B. an empty set.

   C. an equal set.

5. Denominators are a kind of:

   A. product.

   B. number.

   C. sum.

6. Which of the following is <u>always</u> true about an equation using the common factor property?

   A. The equation uses a plus sign but not a multiplication sign.

   B. The equation uses a multiplication sign but not a plus sign.

   C. The equation uses both a multiplication sign and a plus sign.

7. A remainder is a kind of:

   A. fraction.

   B. operation.

   C. number.

8. A mixed fraction is:

   A. a whole number plus a fraction.

   B. a number which is less than one.

   C. made up of factors.

9. Which of the following is NOT always true about an equation using the common factor property?

   A. The equation uses 4 as a common factor.

   B. The equation uses a plus sign and a multiplication sign.

   C. The equation uses parentheses.

10. Which of the following is true for some but NOT all solution sets?

   A. They replace the place holder in a problem.

   B. They make an open sentence a true statement.

   C. They contain one number.

11. Any graph is a kind of:

   A. measurement.

   B. picture.

   C. weight.

12. Which equation does NOT use the common factor property?

   A. $(2 \times 5) + (7 \times 5) = (2 + 7) \times 5$

   B. $(3 \times 6) \times 4 = 3 \times (6 \times 4)$

   C. $(8 \times 9) + (8 \times 6) = 8 \times (9 + 6)$

13. One kind of factor is:

   A. a multiplier.

   B. an addend.

   C. a sum.

14. A place holder is a kind of:

    A. statement.

    B. symbol.

    C. number.

15. Which equation uses the common factor property?

    A. $(4 \times 5) \times 6 = (5 \times 6)$

    B. $(2 \times 8) = (8 \times 2)$

    C. $(7 \times 3) + (9 \times 3) = (7 + 9) \times 3$

16. Kinds of equivalent sets are:

    A. pointed sets.

    B. two sets with a different number of members.

    C. equal sets.

17. What is true for some but NOT all equal sets?

    A. They are made up of two or more sets.

    B. They have the same members.

    C. They are empty.

18. The equation $(6 \times 4) + (5 \times 4) = (6 + 5) \times 4$ uses:

    A. the common factor property.

    B. the associative property.

    C. the commutative property.

19. The number 3 in 2 x 3 = 6 is

A. part of a product.

B. part of a quotient.

C. part of a sum.

20. Which number in 16,785,002 has a place value of one thousand?

A. 5

B. 2

C. 5,002

21. The equation (3 x 5) + (3 x 9) = 3 x (5 + 9) does NOT use

A. the common factor property.

B. the associative property.

C. addends and factors.

22. Any two sets which have the same number of members are called:

A. empty sets.

B. open sets.

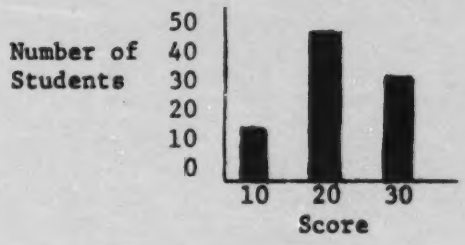
C. equivalent sets.

23. The picture at the right is an example of:

A. a map.

B. a graph.

C. a list of members.



24. An equation using the common factor property is

A. ☆ x (△ + ▱) = (☆ x △) + (☆ x ▱)

B. ☆ + (△ x ▱) = (△ x ▱) + ☆

C. ☆ x (△ x ▱) = (☆ x △) x ▱

25. Which of the following has no end?

A. •

B.

C.

26. In the fraction 3 1/2, the 3 is called:

A. the denominator.

B. the whole number part.

C. the fraction part.

27. Which of the following <u>always</u> uses a plus sign and a multiplication sign in the same equation?

A. the commutative property

B. the common factor property

C. the associative property

28. The ten's place is an example of:

A. a place value.

B. a fraction place.

C. a place holder.

ANSWER SHEET

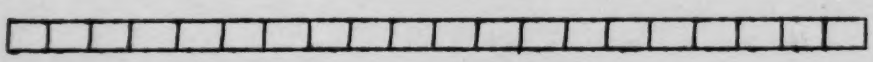
Your Name: _____ Boy/Girl Class Number: _____

Your School: _____ Your Teacher: _____

CIRCLE THE LETTER REPRESENTING THE CORRECT ANSWER FOR EACH QUESTION.

QUESTION	1.	A	B	C	QUESTION	15.	A	B	C
QUESTION	2.	A	B	C	QUESTION	16.	A	B	C
QUESTION	3.	A	B	C	QUESTION	17.	A	B	C
QUESTION	4.	A	B	C	QUESTION	18.	A	B	C
QUESTION	5.	A	B	C	QUESTION	19.	A	B	C
QUESTION	6.	A	B	C	QUESTION	20.	A	B	C
QUESTION	7.	A	B	C	QUESTION	21.	A	B	C
QUESTION	8.	A	B	C	QUESTION	22.	A	B	C
QUESTION	9.	A	B	C	QUESTION	23.	A	B	C
QUESTION	10.	A	B	C	QUESTION	24.	A	B	C
QUESTION	11.	A	B	C	QUESTION	25.	A	B	C
QUESTION	12.	A	B	C	QUESTION	26.	A	B	C
QUESTION	13.	A	B	C	QUESTION	27.	A	B	C
QUESTION	14.	A	B	C	QUESTION	28.	A	B	C

_____ PLEASE DO NOT WRITE BELOW THIS LINE. _____



Directions for Administration of Recognition Level Test

I. The booklets and answer sheets should be distributed to the students with the caution being made that
   a. Students should not open booklets until directed to,
   b. Students should not write on the booklets, and
   c. Students should not begin filling in data on the answer sheet until directed to.

II. Students should be guided in completing the information asked for on the answer sheet.
   a. Students should place both first and last name on the appropriate blank.
   b. Students should place the name of the school on the appropriate blank.
   c. Students should circle 'boy' or 'girl' whichever is appropriate.
   d. For class number, students should place the classroom number on the appropriate blank.
   e. Students' teacher's name should be placed on the appropriate blank.

III. Students should be told that they may now open the booklet to page one.

IV. Ask students to read silently as the directions at the top of page 1 are read aloud. (The directions should be read aloud slowly by the administrator so students can follow.)

V. In working the three examples with the students the following procedures should be followed.

The students should be asked to read silently question 1 while the administrator reads the question and three possible answers aloud to the students.

The administrator should say, "Notice the word 'always' is underlined. The answer must 'always' be true and we should be careful when selecting our answer. A placeholder could be a box as given in answer C, it could be an X as given in answer B, but it must 'always' hold the place for a number. There-

fore, the correct answer is answer A. To show this, we find where 'QUESTION 1' is written on the answer sheet. Beside the words 'QUESTION 1' are the letters A, B, and C. Since A is the correct answer for question 1, a circle should be drawn around the A. Do that now."

(A similar procedure should be continued through question 3.)

VI. After question 3 is completed, comment, "Now we have done the first three questions together. You are to find the correct letter for each of the questions in your booklet just as we have done for the first three questions. You are not to write in the booklet. All answers are to be marked on the answer sheet as we have done for the first three questions. You may now go ahead and complete the other questions in the booklet."

VII. Booklets and answer sheets are to be collected after all students have completed the test. Students who finish quickly should be encouraged to 'check their answers.'

VIII. Students' questions during the testing should be answered tactfully with no clues given. Extreme care should be taken regarding questions concerning the distributive principle.

Part II

Five Four-Item Interview Tests

INTERVIEW TEST 1

Item 1: (18 x 12) + (18 x 19) = (___ x ___)

Item 2: (3 x 7) + (3 x 6) = (___ x ___)

Item 3: (4 x 5) + (7 x 5) = (___ x ___)

Item 4: (16 x 14) + (15 x 14) = (___ x ___)

INTERVIEW TEST 2

Item 1: (9 x 8) + (9 x 3) = (___ x ___)

Item 2: (16 x 19) + (16 x 17) = (___ x ___)

Item 3: (14 x 19) + (18 x 19) = (___ x ___)

Item 4: (5 x 4) + (6 x 4) = (___ x ___)

INTERVIEW TEST 3

Item 1: (8 x 9) + (3 x 9) = (___ x ___)

Item 2: (19 x 16) + (17 x 16) = (___ x ___)

Item 3: (5 x 4) + (5 x 7) = (___ x ___)

Item 4: (14 x 16) + (14 x 15) = (___ x ___)

INTERVIEW TEST 4

Item 1: (12 x 18) + (19 x 18) = (___ x ___)

Item 2: (7 x 3) + (6 x 3) = (___ x ___)

Item 3: (19 x 14) + (19 x 18) = (___ x ___)

Item 4: (4 x 5) + (4 x 6) = (___ x ___)

INTERVIEW TEST 5

Item 1: $(12 \times 18) + (19 \times 18) = (__ \times __)$

Item 2: $(3 \times 7) + (3 \times 6) = (__ \times __)$

Item 3: $(4 \times 5) + (7 \times 5) = (__ \times __)$

Item 4: $(14 \times 16) + (14 \times 15) = (__ \times __)$

$(12 \times 18) + (19 \times 18) = (__ \times __)$

$(3 \times 7) + (3 \times 6) = (___ \times ___)$

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$(4 \times 5) + (7 \times 5) = (__ \times __)$

$(14 \times 16) + (14 \times 15) = (__ \times __)$

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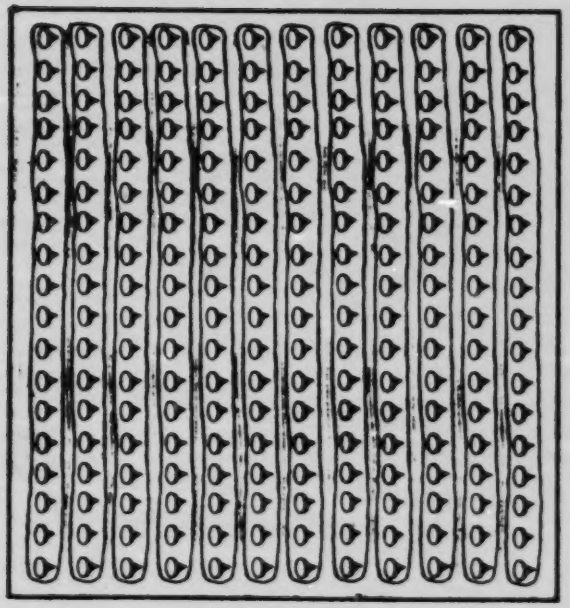
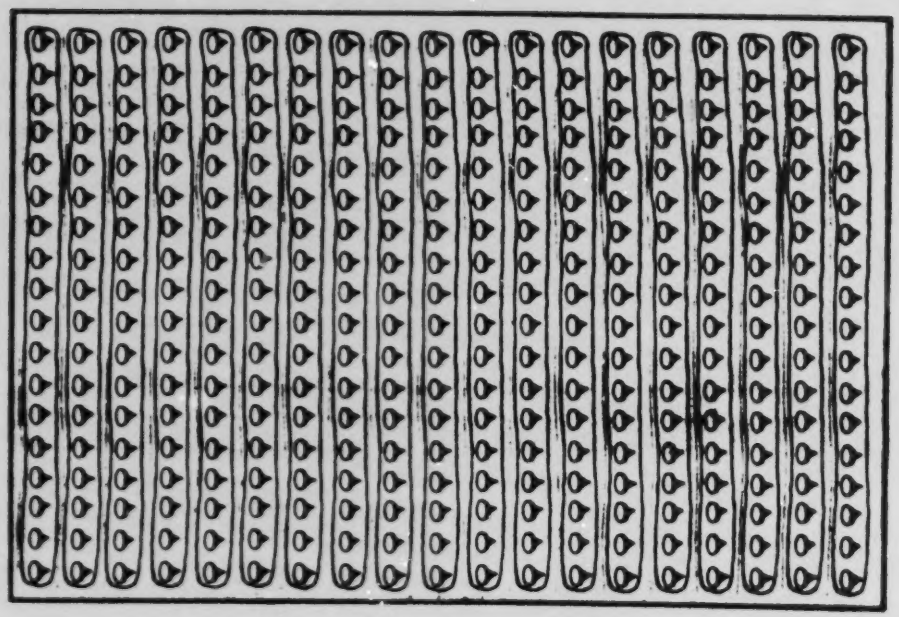
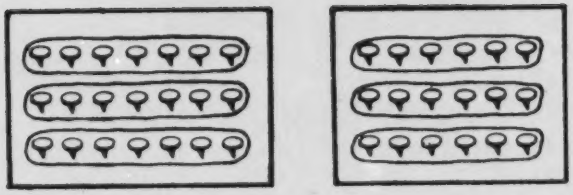
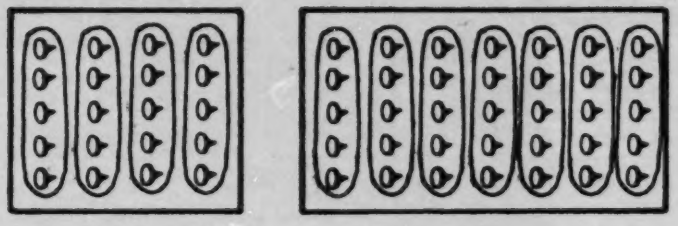




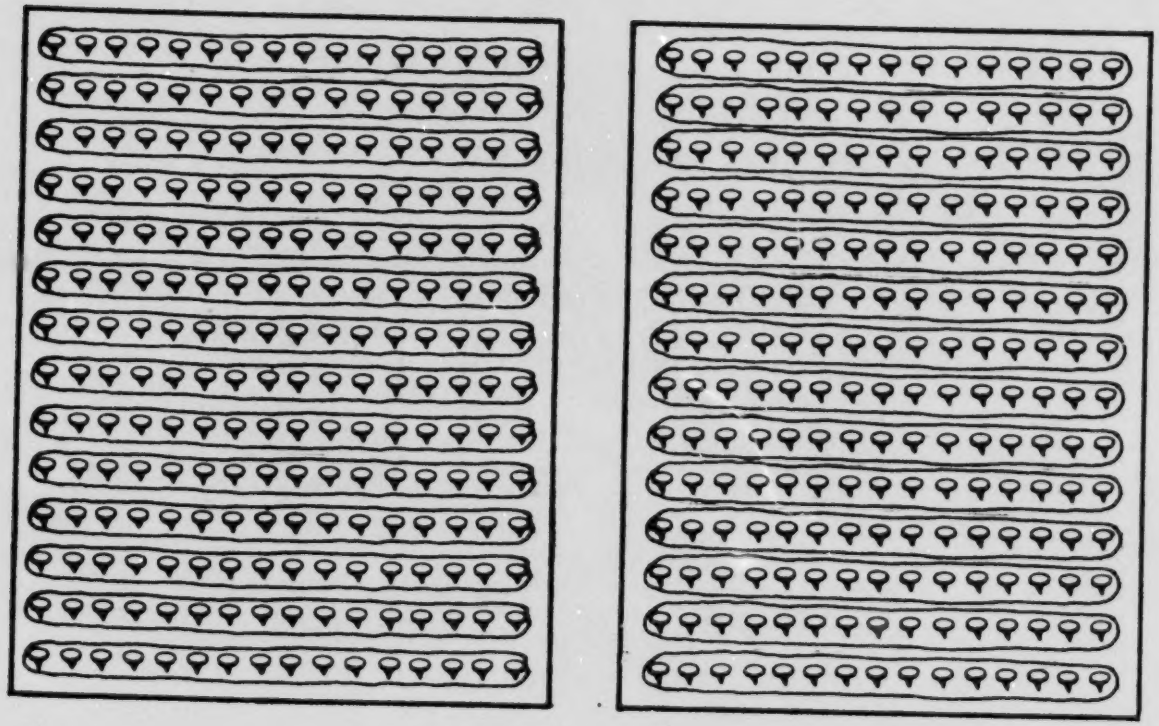
(12 x 18) + (19 x 18) = (___ x ___)



(3 x 7) + (3 x 6) = (___ x ___)



(4 x 5) + (7 x 5) = (___ x ___)



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(14 x 16) + (14 x 15) = (___ x ___)

PART III
INTERVIEW CODING FORM[a]

Student No. _____ Page No. _____

Tape No. _____ Test No. _____ Coder No. _____

****

Prob. Sit. Code: ____ Prob. Pairs: _____ Ans.: _____ Score: _____

DRS: _____ MM(s): _____ Mult. Method: _____ Add. Method: _____

Checked: _____ Context Used: _____ Error (s): _____

Indicates Relation Between: Fac. Method: _____

Q_1 & Set A _____ as desired _____. Error: _____

Q_2 & Set B _____ as desired _____. Error: _____

$Q_1 + Q_2$ & (A U B) _____ as desired _____. Error: _____

Q_3 & (A U B) _____ as desired _____. Error: _____

Const. Diag. of Set A _____ as desired _____. Error: _____

Const. Diag. of Set B _____ as desired _____. Error: _____

Const. Diag. of (A U B) _____ as desired _____. Error: _____

Recognizes Verbalization Cue: _____ Questions &/or Comments: _____

Coder Comments:

****

[a]This is one of 4 identical sections (1 per interview test item) contained on the Interview Coding Form.

DIRECTIONS FOR USING INTERVIEW CODING FORM

Step 1: Place the student number found on the white label of the student's interview packet on the blank adjacent to "Student No."

Example: Student No. 1305

Step 2: Place the tape number found on the box of the recording tape on the blank adjacent to "Tape No."

Example: Tape No. 1-2-045

Step 3: Place the Interview Test number found on the student's interview packet on the blank adjacent to "Test No."

Codes for S, SP, SV, & SPV are 1, 2, 3, & 4 respectively.

Example: Translate 3-SP into 3-3, 2-SPV into 2-4, etc.

Test No. 2-4 (This code is for 2-SPV.)

Step 4: Place the coder's number on the blank adjacent to "Coder No."

Example: Coder NO. 2

Step 5: Determine the Problem Situation Code and place the proper code on the blank adjacent to "Prob. Sit.Code."

Problem Situation Codes:

1....L-H; T.....Left-Horizontal; Small No.
2....L-H; L.....Left-Horizontal; Large No.
3....R-H; T.... Right-Horizontal; Small No.
4....R-H; L.....Right-Horizontal; Large No.

Example: (3 x 5) + (3 x 7) = (x) represents problem situation code 1.

Prob. Sit. Code: 1

Step 6: Determine the pair of numbers in each indicated product of the given open sentence and place the proper pairs on the blank adjacent to "Prob. Pairs."

Example: For the open sentence (3 x 5) + (3 x 7) = (x), the pairs (3,5) and (3,7) in the order given in the open sentence should be placed on the appropriate blank.

Prob. Pairs: (3,5); (3,7)

Step 7: Determine the final 2 factors which the student placed on the blanks for a given open sentence and place the ordered pair of factors on the blank adjacent to "Ans."

Example: (3 x 5) + (3 x 7) = (_9_ x _35_) indicates that the ordered pair (9,35) represents the student's final answer. Therefore, (9,35) should be placed on the appropriate blank.

Ans.: (9,35)

Note: If no answer is indicated or if a number is placed on only one blank, place the number 999 on the blank adjacent to the term "Ans."

Ans.: 999

Step 8: Determine if the answer given by the student is a correct answer. If it is, place a 1 on the blank adjacent to the term "Score." If not, place a 0 on the blank adjacent to the term "Score."

Example: (3 x 5) + (3 x 7) = (_9_ x _35_) is incorrect.

Score: 0

Step 9: Determine if the student was asked to reread the open sentence and place the appropriate coding on the blank adjacent to the term "DRS" (i.e., Difficulty in Reading Sentence). A 2-digit code is to be used for coding the reading difficulty. The first digit represents the number of times the student reads the sentence (including the first time when the student is asked to read the open sentence). This means that each student should read each open sentence at least once. The second digit represents the type of difficulty the student had in reading the sentence. The codes for this latter portion are:

0---No difficulty noted
1---Difficulty reading placeholders (i.e., the blanks)
2---Missed a number (skipped it)
3---Missed an operation sign (skipped it)
4---Missed an equal sign (skipped it)
5---Misread a number
6---Misread an operation sign (either the + or x)
7---Misread an equal sign
8---Other than 0-7
9---A combination of 0-8

Examples: If a student read the open sentence correctly the first time, the following code would result.

DRS: 1-0

If the student missed an operation sign (skipped it) and reread it correctly the second time, the following code would result.

DRS: 2-3

If the student missed an operation sign the first time, misread it the second time, then read it correctly the third time, the following code would result.

DRS: 3-9 (3,6)

Notes:

A. Following the use of a 9 code, the individual error codes are to be sequenced in parenthesis as indicated in the last example above.
B. If the student read or began to read an open sentence incorrectly at first and corrected himself, the code 2-0 is to be given.
C. If the student was not asked to read an open sentence, the code 0-0 is to be given.
D. The student's reading of the open sentence is to be coded at the introduction of each open sentence and after the completion of each open sentence with the codings separated by a semicolon.

Example: If the student reads the open sentence correctly at the introduction of an open sentence and after working the open sentence, the following code would result.

DRS: 1-0; 1-0

Step 10: The mathematical method used by the student in attempting to find a solution to the open sentence presented should be determined and the code representing that method should be placed on the blank adjacent to "MM(s)." Codes for different methods are:

1....The student used the distributive principle.
2....The student found the sum of the two indicated products and then factored the sum using the factors as a solution pair.
3....The student found the two indicated products and used the two products as a solution pair.
4....The student added the common factor to the common factor to find a sum, added the other two factors to find a second sum, then used these two sums as a solution pair.
5....The student added the two factors in the first indicated product, added the two factors in the second indicated product, and used these two sums as a solution pair.
6....The student tried but gave up or went to next open sentence - no method indicated.
7....The student did not try.
8....Other than 1-7. (A brief note in the section for coder comments should indicate what procedure was used.)

Notes:

A. If the student made an error when using a method, the method should still be determined as if the error had not been made.

B. If a student began to use a method then changed to another method, the last method used should be the one coded.

Example: If the student multiplied three times three and five times seven to find the answer to the open sentence in Step 8, the following code would result.

MM(s): 8

Step 11: Each open sentence contains two indicated products. Many students may attempt to find these products. The multiplication method used in finding each product should be determined and the appropriate 2-digit code (A digit for each indicated product) should be placed in the blank adjacent to "Mult. Method." If the student did not attempt to find the indicated products, leave the blank empty. If the student attempted to find only one of the indicated products a one digit code may be used.

The codes for use are:

1....Standard Algorithm
2....Recall of Basic Facts
3....Repeated Addition (No sign of grouping when adding)
4....Repeated Addition (Grouped when adding)
5....Other than basic fact computed mentally
6....Other than 1-5.

Example: (13 x 12) + (13 x 10) = (x)

If a student uses the standard algorithm to compute the first indicated product and computes the second indicated product mentally, the following code would result

Mult. Method: 1;5

Note: Even if an error was made, the method used by the student should be determined as if the error had not been made. Other multiplications (not the indicated products) are not to be coded.

Step 12: Each open sentence contains an indicated addition. Many students may attempt to find the sum of the two indicated products. The addition method used in finding this sum should be determined and the appropriate code should be placed on the blank adjacent to "Add. Method." If the student did not attempt to find the indicated sum, leave the blank empty.

The codes for use are:

1....Counting from 1
2....Counting on (The student begins counting by 1 following the first or second addend.)

3....Grouping
4....Recall of basic facts
5....Standard Algorithm
6....Other than basic fact mentally computed
7....Other than 1-6

Example: (3 x 5) + (3 x 7) = (__x__)

If a student began to add by counting 15, 16, 17,...,35 the following code would result.

Add. Method: __2__

Note: Even if an error was made, the method used by the student should be determined as if the error had not been made. Other additions (not the indicated sum) are not to be coded.

Step 13: If the student attempted to factor the sum of the two indicated products, the method used in attempting to factor should be determined and the code representing that method should be placed on the blank adjacent to "Fac. Method". If the student did not attempt to factor the sum of the two indicated products, the blank should be left empty.

The codes for use are:

1---Recall of basic facts
2---Estimated and used division to check
3---Estimated and used multiplication to check
4---Estimated and did not check
5---Used a method based on the number which is to be factored (e.g., dividing by 2 if an even number is to be factored and the student indicated use of the "evenness" of the number when choosing 2 as a factor)
6---Used the distributive property to find both factors after adding the two indicated products.
7---Used the common factor and divided to find the other factor after adding the two indicated products.
8---Gave up trying to factor the sum of the two indicated products and went on to the next open sentence
9---Method other than 1-8

Note: The method used should be determined regardless if an error was or was not made by the student. Note that a combination of methods could result. If a combination of methods was used, the codes representing the methods should be sequenced on the appropriate blank with commas between the codes. A comment by the coder clarifying the sequence of methods considered by the student may be needed.

Example: (3 x 4) + (3 x 6) = (5 x 6)

If a student found the sum of the two indicated products and recalled the basic fact as indicated above, the following code would result.

Fac. Method: 1

Step 14: If the student checked the solution found, determine which code below represents the method used and place the corresponding number code on the blank adjacent to "Checked." If the student failed to check the solution, leave the blank empty.

The codes for use are:

1---Retraced steps
2---Checked for reasonableness of answer
(Coder should comment on how reasonableness was checked)
3---Changed methods when checking (Coder should comment on the change made).
4---Other than 1-3 (Coder should make brief comment)

Example: If a student used the distributive property to find an answer, then the student found the sum of the two indicated products and checked if the product of the two selected factors equaled the sum, the following code would result.

Checked: 3

Step 15: If the student showed signs of using a definite context or a combination of contexts, indicate by sequencing the number codes below on the blank adjacent to "Context Used."

The codes for use are:

1---Symbolic
2---Pictorial (the sets were used in finding the solution)
3---Verbal Cue (the term distributive property or common factor rule was instrumental in the students choice of method used in finding the solution).

Example: If a student used the mathematical method coded number 2 without using the pictorial or the verbal cues, the following code would result.

Context Used: 1

Step 16: If an error was made by the student, determine which error code below best represents the error and place the corresponding number-letter code on the blank adjacent to "Error(s)." If more than one error was made, place the sequenced codings separated by a comma on the appropriate blank. If no error was made, leave the blank empty.

The codes for use are:

1---Multiplication

a.---The ones digit was multiplied by the ones digit and the tens digit by the tens digit (may have used the tens digit as a multiple of ten as in the second example below).

Examples:

$$\begin{array}{r} 17 \\ \times\ 16 \\ \hline 142 \end{array} \qquad \begin{array}{r} 13 \\ \times\ 12 \\ \hline 106 \end{array}$$

b.---The ones digit was multiplied by the ones digit and the tens digit by the tens digit, adding the carried number from the ones digit multiplication.

Example:

$$\begin{array}{r} {\scriptstyle 1\,\prime} \\ 27 \\ \times\ 32 \\ \hline 74 \end{array}$$

c.---The carried number was not included in the partial product.

Example:

$$\begin{array}{r} {\scriptstyle 2} \\ 17 \\ \times\ 3 \\ \hline 31 \end{array}$$

d.---The place value of partial products was confused.

Examples:

$$\begin{array}{r} 13 \\ \times\ 20 \\ \hline 00 \\ 26 \\ \hline 26 \end{array} \qquad \begin{array}{r} 21 \\ \times\ 11 \\ \hline 21 \\ 21 \\ \hline 42 \end{array}$$

e.---When carrying, the carried number was added to the digit in the factor and placed in the answer or added incorrectly in some manner when finding the answer or partial products.

Examples:

$$\begin{array}{r} {\scriptstyle 1\,\prime} \\ 13 \\ \times\ 4 \\ \hline 22 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,\prime} \\ 13 \\ \times\ 4 \\ \hline 82 \end{array}$$

f.---One of the digits in the multiplier was not used in finding the product.

Example:
$$\begin{array}{r} 23 \\ \times\ 17 \\ \hline 161 \end{array}$$

g.---An error was made in adding the partial products.

Example:
$$\begin{array}{r} 23 \\ \times\ 17 \\ \hline 161 \\ 23 \\ \hline 291 \end{array}$$

h.---The student multiplied by repeated addition or counting with an error in the addition or counting.

Example:
$$\begin{array}{r} 7 \\ \times\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ 7 \\ 7 \\ \hline 19 \end{array}$$

i.---Other than a-h or j-l.

j.---Error in multiplication when finding products other than the indicated products.

k.---Recall of a basic multiplication fact incorrectly.

l.---The student added when supposed to multiply.

2--- Addition

a.---Basic addition combination recalled incorrectly.

b.---When counting was used, the student lost count of the counting.

c.---Failure to add the carried digit even when written above the top digit in column to the left.

Example:
$$\begin{array}{r} {\scriptstyle 1} \\ 15 \\ 17 \\ \hline 92 \end{array}$$

d.---Wrong digit was carried from the sum of one column to the next.

Example:
$$\begin{array}{r} {\scriptstyle 2\ \ \ } \\ 15 \\ 17 \\ \hline 42 \end{array}$$

e.---Intending to add, a student multiplied.

Example:
$$\begin{array}{r} 13 \\ 7 \\ \hline 91 \end{array}$$

f.---While using grouping in adding, an error in grouping was made (e.g., lost track of number of groups).

g.---Other than a-f or h.

h.---The student made an error while calculating mentally (i.e., error done while doing the addition in one's mind without writing the calculations down.).

3.---Factoring

4.---Operation order confused (e.g., The student paid no attention to the order imposed on the operations by parentheses).

5.---Plus and multiplication sign confusion (Note that a definite sign of confusion should be indicated by the student. The use of method 3 as a procedure to solve an open sentence is not to be interpreted as this type error unless so indicated by the student.)

6.---Error due to use of sets in finding a solution pair

7.---Error due to misinterpretation of verbal cue

8.---Error in use of the distributive property

Example: $(3 \times 7) + (3 \times 6) = (\underline{3} \times \underline{42})$

9.---Error in writing when moving numerals from one place to another when completing an open sentence.

10.---Other than 1-9 or couldn't determine the error made.

Step 17: If the student indicated a relation between the indicated products (Q_1 and/or Q_2) and the sets representing these indicated products after the open sentences are presented in the SP or SPV context, or if the student tried to construct diagrams of sets, or recognized and tried to use the verbalization cue, the coder should indicate this procedure by checking the appropriate blanks. If the connections are ones desired (e.g., $\underline{a}$ sets of $\underline{b}$ and $\underline{a}$ x $\underline{b}$), a check should be placed on the "as desired" blank. If an error was made, the coder should briefly describe the error on the blank immediately to the right of the term "error." Blanks should be left empty if no information is given by the taped interview.

Step 18: If the student raised questions or made comments, these should be indicated by sequencing Q's for questions and C's for comments on the blank adjacent to "Questions &/or Comments." If no questions are raised or comments made by the student which the coder deems important, the blank should be left empty.

Example: The coding below indicates that the student asked 2 questions, made a comment, asked a question, then made a comment. Note that these questions and comments are not all comments and questions made by the student. These questions and comments are those which the coder believes may be important to the investigation. A brief explanation concerning the questions and comments coded should be made by the coder in the space for "Coder Comments."

Questions &/or Comments: Q, Q, C, Q, C

Step 19: Perhaps, one of the most important items to be completed on the coding form is the space left for "Coder Comments." If the coder notes any irregularities, any interesting insights, or any interesting communication given by the student either in writing or in talking, then this space should be used for recording these items or any other items the coder thinks important to the investigation.

PART IV

OPPORTUNITY TO LEARN SURVEY

During the past few weeks, a group of fifth-grade students from the La Follette Attendance Area has been participating in an investigation concerning elementary school students' recognition and use of the distributive property (common factor rule). To help interpret the students' responses which were made during the investigation, certain background information is desirable. Since you may have taught some of the participating students, certain information can be provided by you better than by anyone else.

/Note that this form is to be completed anonymously as this survey is to be used only in determining the group of students' (not individual student's) opportunity to learn certain topics which may be related to students' recognition and use of the distributive property (common factor rule)./

Your cooperation is needed in this particular phase of the study and is deeply appreciated.

Third-grade teachers would have taught these students during the academic year 1970-71, fourth-grade teachers would have taught these students during the academic year 1971-72, and fifth-grade teachers would be teaching these students during the current academic year. The following information should be given according to the academic year the students were taught.

During which grade level did you teach these students? _____.

Was the Houghton-Mifflin text used as the basic material from which the students were taught the distributive idea? _____.

If not, please indicate which basic material was used.

_____.

Circle the expression below which best represents the fractional part of the total amount of material presented on distributivity by Houghton-Mifflin during this grade level which was presented to the students.

1/4 to 1/2

1/2 to 2/3

more than 2/3

Was any additional material on distributivity presented? _____.

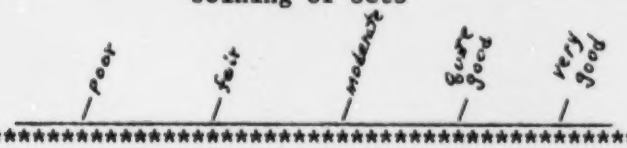
If so, please describe. _____

_____.

Place an "x" on each continuum below so that the position of the "x" best describes the students' opportunity to learn the indicated item by the end of the school year when taught. Note that this does not mean the students learned the item.

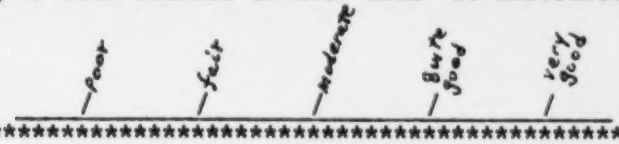
****

Joining of sets



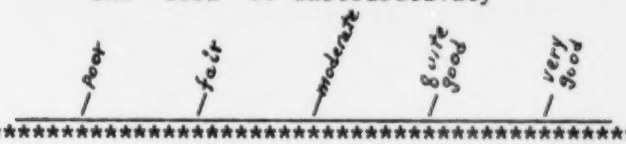
****

Joining of sets related to the "idea" of distributivity



****

The "idea" of distributivity

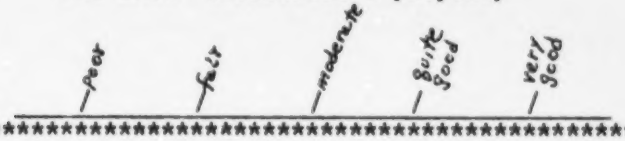


****

The term "common factor rule"

****

The term "distributive property"

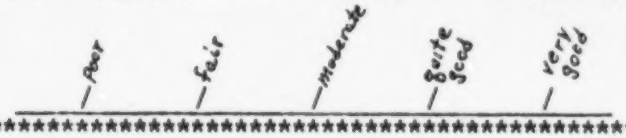


****

The "use" of distributivity

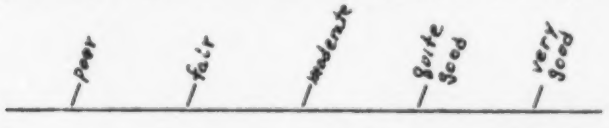
****

Equations like (3 x 4) + (3 x 5) = 3 x (4 + 5)



****

Equations like (3 x 4) + (3 x 5) = (3 x 9)



APPENDIX B

RAW DATA

TABLE B-1

Correct Solutions by Student, Context, and Interview Test Item

Student number	Context level	Interview test items				Total correct solutions
		L-H;T	L-H;L	R-H;T	R-H;L	
09	S	0	0	0	0	0
19	S	1	1	1	1	4
13	S	1	0	1	0	2
04	S	0	0	0	0	0
03	S	0	0	0	0	0
10	SP	1	0	1	0	2
11	SP	1	1	1	1	4
12	SP	0	0	0	0	0
16	SP	1	1	1	1	4
02	SP	0	0	0	0	0
14	SV	0	1	0	1	2
07	SV	0	0	0	0	0
15	SV	0	0	1	0	1
17	SV	1	1	1	1	4
06	SV	0	0	0	0	0
08	SPV	0	0	0	0	0
20	SPV	0	0	0	0	0
18	SPV	0	0	0	0	0
01	SPV	0	0	0	0	0
05	SPV	0	0	0	0	0
	Total	6	5	7	5	23

S: symbolic
SP: symbolic pictorial
SV: symbolic and verbal
SPV: symbolic, pictorial, and verbal.

L-H: left-horizontal
R-H: right-horizontal
L: large number size
T: small number size
0: incorrect solution
1: correct solution

TABLE B-1
(Con't.)

Student number	Context level	Interview test items				Total correct solutions
		L-H;T	L-H;L	R-H;T	R-H;L	
27	S	1	1	1	1	4
26	S	0	0	0	0	0
34	S	1	1	1	1	4
22	S	0	0	0	0	0
28	S	0	1	1	0	2
40	SP	0	0	0	0	0
29	SP	0	0	0	0	0
37	SP	0	0	0	0	0
36	SP	0	0	0	0	0
33	SP	0	0	0	0	0
38	SV	0	0	0	0	0
32	SV	0	0	0	0	0
31	SV	0	0	0	0	0
30	SV	1	1	1	1	4
35	SV	0	0	0	0	0
24	SPV	0	0	0	0	0
39	SPV	0	0	0	0	0
21	SPV	0	0	0	0	0
25	SPV	0	0	0	0	0
23	SPV	0	0	0	0	0
	Total	3	4	4	3	14

TABLE B-1
(Con't.)

Student number	Context level	Interview test items				Total correct solutions
		L-H;T	L-H;L	R-H;T	R-H;L	
50	S	0	0	0	0	0
45	S	0	0	0	0	0
53	S	0	0	0	0	0
43	S	0	0	0	0	0
47	S	0	0	0	0	0
44	SP	0	0	0	0	0
52	SP	1	1	1	1	4
48	SP	1	0	0	0	1
59	SP	0	0	0	0	0
41	SP	0	0	0	0	0
57	SV	0	0	0	0	0
60	SV	0	0	0	0	0
54	SV	1	1	1	1	4
56	SV	0	0	0	0	0
49	SV	0	0	0	0	0
55	SPV	0	0	0	0	0
51	SPV	0	0	0	0	0
46	SPV	0	0	0	0	0
42	SPV	0	0	0	0	0
58	SPV	0	0	0	0	0
	Total	3	2	2	2	9

TABLE B-1
(Con't.)

Student number	Context level	Interview test items				Total correct solutions
		L-H;T	L-H;L	R-H;T	R-H;L	
74	S	0	0	0	0	0
63	S	0	0	0	0	0
76	S	0	0	0	0	0
77	S	0	0	0	0	0
80	S	0	1	1	0	2
61	SP	0	0	0	1	1
69	SP	0	0	0	0	0
68	SP	0	1	0	0	1
66	SP	0	0	0	0	0
65	SP	0	0	0	0	0
78	SV	0	0	0	0	0
62	SV	1	1	1	1	4
64	SV	0	0	0	0	0
79	SV	0	0	0	0	0
67	SV	0	0	0	0	0
70	SPV	0	0	1	0	1
73	SPV	0	0	0	0	0
72	SPV	0	0	0	0	0
71	SPV	0	0	0	0	0
75	SPV	0	0	0	0	0
	Total	1	3	3	2	9

TABLE B-2

Solution Methods[a] by Student, Context, and Test Item

Student number	Context level	Solution Method: Interview test items			
		L-H;T	L-H;L	R-H;T	R-H;L
09	S	1	1	1	1
19	S	1	1	1	1
13	S	2	2	2	2
04	S	3	3	3	3
03	S	3	3	3	3
10	SP	2	8	2	3
11	SP	1	1	1	1
12	SP	3	3	3	3
16	SP	1	1	1	1
02	SP	3	3	3	3
14	SV	3	1	3	1
07	SV	3	3	3	3
15	SV	4	5	1	8
17	SV	1	1	1	1
06	SV	3	3	3	3
08	SPV	3	3	3	3
20	SPV	3	3	3	3
18	SPV	3	3	3	3
01	SPV	3	3	3	3
05	SPV	3	3	3	3

[a]See Table 26 in Chapter V for identification of solution method codes.

TABLE B-2
(Con't.)

Student number	Context level	Solution Method: Interview test items L-H;T	L-H;L	R-H;T	R-H;L
27	S	2	2	2	2
26	S	3	3	3	3
34	S	1	1	1	1
22	S	3	3	3	3
28	S	2	2	2	2
40	SP	3	3	3	3
29	SP	3	3	3	3
37	SP	3	3	3	6
36	SP	3	3	3	6
33	SP	3	3	3	3
38	SV	3	3	3	3
32	SV	3	3	3	3
31	SV	3	3	3	3
30	SV	1	1	1	1
35	SV	3	3	3	3
24	SPV	3	3	3	3
39	SPV	3	3	3	3
21	SPV	3	3	3	3
25	SPV	3	3	3	3
23	SPV	4	4	4	4

TABLE B-2
(Con't.)

Student number	Context level	Solution Method: Interview test items			
		L-H;T	L-H;L	R-H;T	R-H;L
50	S	3	3	3	3
45	S	3	3	3	3
53	S	3	8	8	8
43	S	3	4	3	4
47	S	3	3	3	3
44	SP	8	8	8	8
52	SP	1	1	1	1
48	SP	2	2	2	2
59	SP	3	3	3	3
41	SP	3	3	3	3
57	SV	8	8	8	8
60	SV	3	3	3	3
54	SV	1	1	1	1
56	SV	3	3	3	3
49	SV	3	3	3	3
55	SPV	6	6	6	6
51	SPV	8	8	8	8
46	SPV	3	3	3	3
42	SPV	3	3	3	3
58	SPV	3	3	3	3

TABLE B-2
(Con't.)

Student number	Context level	Interview test items			
		L-H;T	L-H;L	R-H;T	R-H;L
74	S	3	6	3	6
63	S	5	5	5	4
76	S	1	1	1	1
77	S	3	3	8	8
80	S	8	8	8	8
61	SP	1	8	1	1
69	SP	3	3	3	3
68	SP	2	2	2	2
66	SP	3	3	3	3
65	SP	3	3	3	3
78	SV	3	3	3	3
62	SV	2	2	2	2
64	SV	3	3	3	3
79	SV	3	3	3	3
67	SV	3	3	3	3
70	SPV	2	2	2	2
73	SPV	3	6	3	6
72	SPV	3	3	3	3
71	SPV	3	3	3	3
75	SPV	3	3	3	3

APPENDIX C

CHARACTERISTICS OF STUDENT SAMPLE

TABLE C

Characteristics of Students[a]

Student	School attended	IQ	Math. ach.	Concept attain.	Sex
01	1	120	32	14	M
02	1	87	34	12	M
03	1	107	26	7	M
04	1	111	36	13	M
05	1	143	34	12	M
06	1	104	16	9	M
07	1	129	40	16	F
08	1	124	27	7	F
09	1	127	40	14	F
10	1	100	28	12	F
11	2	118	43	13	F
12	2	140	27	9	F
13	2	113	35	12	F
14	3	111	34	12	F
15	3	95	14	6	F
16	3	114	38	16	M
17	3	124	39	11	M
18	3	92	19	6	F
19	3	100	25	7	F
20	3	107	27	13	F

[a]Tests from which IQ, mathematics achievement, and concept attainment data were obtained are described in Chapter V.

TABLE C
(Con't.)

Student	School attended	IQ	Math. ach.	Concept attain.	Sex
21	1	99	16	9	M
22	1	119	19	10	F
23	1	157	40	14	F
24	1	82	19	9	M
25	1	101	24	10	F
26	1	113	34	12	M
27	1	120	36	10	M
28	2	114	32	11	F
29	2	96	19	8	M
30	2	126	37	9	F
31	2	121	31	13	M
32	2	111	17	9	M
33	3	117	15	7	F
34	3	124	36	12	M
35	3	94	26	10	F
36	3	116	33	14	F
37	3	85	24	9	M
38	3	108	23	10	M
39	3	119	41	13	M
40	3	100	31	12	M

TABLE C
(Con't.)

Student	School attended	IQ	Math. ach.	Concept attain.	Sex
41	1	118	33	14	M
42	1	118	41	10	M
43	1	100	15	8	M
44	1	102	27	12	F
45	1	100	30	9	F
46	1	124	21	12	F
47	1	102	21	6	M
48	2	106	33	11	F
49	2	105	15	7	M
50	2	116	23	11	F
51	2	80	14	8	F
52	3	104	34	13	F
53	3	91	16	6	F
54	3	100	24	8	F
55	3	100	16	7	F
56	3	109	18	9	M
57	3	102	15	9	F
58	3	114	31	13	M
59	3	106	31	10	M
60	3	114	37	13	F

TABLE C
(Con't.)

Student	School attended	IQ	Math ach.	Concept attain.	Sex
61	1	112	37	8	M
62	1	97	34	10	M
63	1	94	39	12	M
64	1	131	29	13	F
65	1	95	13	8	F
66	2	112	27	10	F
67	2	124	19	7	F
68	2	112	30	11	F
69	2	104	20	7	M
70	2	97	27	10	M
71	3	98	24	7	F
72	3	102	21	8	F
73	3	88	20	10	M
74	3	111	23	2	M
75	3	134	26	10	F
76	3	109	24	8	F
77	3	108	17	9	F
78	3	125	35	11	M
79	3	112	27	12	F
80	3	81	21	11	F

APPENDIX D

DATA FROM THE RELIABILITY STUDY OF THE CODING OF STUDENTS' PERFORMANCE ON INTERVIEW TESTS

TABLE D-1

Results of Interview Data Coding During Reliability Study

Student	Coder[a]	Category coded[b]										
		PSC	Score	DRS (before)	DRS (after)	MM	Mult. method	Add. method	Fac. method	Checked	Context used	Errors
45	1	1	0	2-1	1-0	3	(2,2)	—[c]	—	—	1	—
	2	1	0	2-1	1-0	3	(2,2)	—	—	—	1	—
64	1	3	0	1-0	1-0	3	(2,2)	—	—	—	1	—
	2	3	0	1-0	1-0	3	(2,2)	—	—	—	1	—
07	1	2	0	2-5	1-0	3	(1,1)	—	—	—	1	—
	2	2	0	2-5	1-0	3	(1,1)	—	—	—	1	—
46	1	2	0	1-0	1-0	3	(6,6)	—	—	—	1	1-b; 1-b
	2	2	0	1-0	1-0	3	(1,1)	—	—	—	1	1-b; 1-b
61	1	2	0	1-0	2-0	8	—	—	—	—	1	9
	2	2	0	1-0	2-0	8	—	—	—	—	1	—
02	1	1	0	1-0	1-0	3	(2,2)	—	—	—	1	—
	2	1	0	2-0	1-0	3	(2,2)	—	—	—	1	—

[a]Coder 1 — author; coder 2 -- graduate student

[b]Categories and abbreviations are explained in the directions for use of the coding form located in Appendix A, Part III. Categories not appearing in Table D-1 were not used by students enough times to warrant recording.

[c]Three dashes represent student's lack of use.

TABLE D-1
(continued)

Student	Coder	PSC	Score	DRS (before)	DRS (after)	MM	Category coded: Mult. method	Add. method	Fac. method	Checked	Context used	Errors
59	1	3	0	1-0	1-0	3	(2,2)	---	---	---	1	---
	2	3	0	1-0	1-0	3	(2,2)	---	---	---	1	---
80	1	3	1	1-0	1-0	8	---	---	---	---	1	---
	2	3	1	1-0	1-0	1	---	---	---	---	1	---
18	1	4	0	1-0	1-0	3	(1,1)	---	---	---	1	---
	2	4	0	1-0	1-0	3	(1,1)	---	---	---	1	---
38	1	3	0	1-0	2-0	3	(2,2)	---	---	---	1	---
	2	3	0	1-0	2-0	3	(2,2)	---	---	---	1	---
54	1	4	1	1-0	1-0	1	---	---	---	---	1	---
	2	4	1	1-0	1-0	1	---	---	---	---	1	---
19	1	2	1	1-0	1-0	1	---	---	---	---	1	---
	2	2	1	1-0	1-0	1	---	---	---	---	1	---
30	1	1	1	1-0	1-0	1	---	---	---	---	1	---
	2	1	1	1-0	1-0	1	---	---	---	---	1	---

TABLE D-1
(continued)

Student	Coder	Category coded										
		PSC	Score	DRS (before)	DRS (after)	MM	Mult. method	Add. method	Fac. method	Checked	Context used	Errors
22	1	2	0	2-5	1-0	3	(1,1)	—	—	—	1	—
	2	2	0	2-5	1-0	3	(1,1)	—	—	—	1	—
23	1	1	0	1-0	1-0	4	—	—	—	—	1	—
	2	1	0	1-0	1-0	4	—	—	—	—	1	—
73	1	3	0	1-0	1-0	3	(2,2)	—	—	—	1,2	—
	2	3	0	1-0	1-0	3	(2,2)	—	—	—	1,2	—
53	1	1	0	1-0	1-0	3	(2,2)	—	—	—	1	—
	2	1	0	1-0	1-0	3	(2,2)	—	—	—	1	—
16	1	3	1	2-6	1-0	1	—	—	—	—	1	—
	2	3	1	2-6	1-0	1	—	—	—	—	1	—
77	1	4	0	1-0	2-7	8	(1,1)	—	—	—	1	—
	2	4	0	1-0	2-7	8	(1,1)	—	—	—	1	1-ℓ; 1-ℓ
20	1	3	0	1-0	1-0	3	(2,2)	—	—	—	1	—
	2	3	0	1-0	1-0	3	(2,2)	—	—	—	1	—

TABLE D-1
(continued)

Student	Coder	PSC	Score	DRS (before)	DRS (after)	MM	Category coded: Mult. method	Add. method	Fac. method	Checked	Context used	Errors
15	1	1	0	1-0	1-0	4	---	---	---	---	1	---
	2	1	0	1-0	1-0	4	---	---	---	---	1	---
24	1	2	0	3-5	1-0	3	(1,1)	---	---	---	1	1-k; 1-g
	2	2	0	2-5	1-0	3	(1,1)	---	---	---	1	1-k; 1-g
62	1	1	1	1-0	1-0	2	(2,2)	6	7	---	1	---
	2	1	1	1-0	1-0	2	(2,2)	---	1	---	1	---
70	1	3	1	1-0	1-0	2	(2,2)	5	3	---	1	---
	2	3	1	1-0	1-0	2	(2,2)	5	1	---	1	---
09	1	1	0	1-0	1-0	1	---	---	---	---	1	---
	2	1	0	1-0	1-0	8	---	---	---	---	1	---
48	1	3	0	1-0	1-0	2	(2,2)	6	3	---	1	2-h; 1-k
	2	3	0	1-0	2-0	2	(1,1)	---	3	---	1	2-h; 1-k
37	1	2	0	1-0	2-6	3	(1,1)	---	---	---	1	1-i
	2	2	0	1-0	2-6	3	(1,1)	---	---	---	1	1-k

TABLE D-1
(continued)

Student	Coder	Category coded										
		PSC	Score	DRS (before)	DRS (after)	MM	Mult. method	Add. method	Fac. method	Checked	Context used	Errors
27	1	1	1	1-0	1-0	2	(2,2)	5	6	3	1	—
	2	1	1	1-0	1-0	2	(2,2)	5	3	—	1	—
68	1	4	0	1-0	1-0	2	(1,1)	5	—	—	1,2	1-e; 2-e
	2	4	0	1-0	1-0	8	(1,1)	—	—	—	1	1-e; 2-e
40	1	1	0	1-0	1-0	3	(2,2)	—	—	—	1	—
	2	1	0	1-0	1-0	3	(2,2)	—	—	—	1	—
55	1	3	0	1-0	0-0	6	(2,2)	—	—	—	1	—
	2	3	0	1-0	0-0	8	—	—	—	—	1	—
57	1	4	0	1-0	1-0	8	(1,1)	—	—	—	1	1-k; 1-a; 1-a
	2	4	0	1-0	1-0	8	(1,1)	—	—	—	1	1-a; 1-a
% agreement between coder 1 and coder 2		100	100	100[d]	100[d]	87.5	90.6	90.6	90.6	96.9	96.9	84.6

[d] % agreement is based on error classification rather than number of times an open sentence was read.

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